ABHYAS 02 (2024) / XL

Class 11 - Mathematics

1. If f(x) = log $\left(\frac{1+x}{1-x}\right)$ and g(x) = $\frac{3x+x^3}{1+3x^2}$ Then f(g)(x) is equal to

   a) f (3x) b) -f (x)

   c) $[f(x)]^3$ d) 3 f(x)

2. Let A = {1, 2, 3} and consider the relation R = {1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1,3)}. Then R is

   a) neither symmetric, nor transitive b) symmetric and transitive

   c) reflexive but not symmetric d) reflexive but not transitive

3. If $[x]^2$ - 5[x] + 6 = 0, where [.] denote the greatest integer function, then

   a) x∈[2, 4) b) x∈(2, 4]

   c) x∈[3, 4] d) x∈[2, 4]

4. Let f : R → R, defined by f(x) = $\begin{cases} 1, \text{ if } x \in Q \\ -1, \text{ if } x \notin Q \end{cases}$. Find: f($\pi$)

5. Let R be a relation on N × N defined by (a,b) R (c, d) ⇔ a + d = b + c for all (a, b), (c, d) ∈ N × N Show that: (a, b) R (c,d) ⇒ (c, d) R (a, b) for all (a, b), (c, d) ∈ N × N.

6. Find the domain and the range of the real function: f(x) = $\frac{ax-b}{cx-d}$

7. Define a relation R on the set N of natural numbers by R = {(x, y): y = x + 5, x is a natural number less than 4; x, y ∈ N]. Depict this relationship using roster from. Write down the domain and the range.

8. Let A = R – {3} and B = R- {1}. Consider the function of f: A → B defined by f(x) = $\frac{x-2}{x-3}$ is one – one and onto.

9. The product of first n odd terms of a G.P. whose middle term is m is

   a) none of these b) m^n

   c) n^m d) mn

10. If x = $\left(a + \frac{a}{r} + \frac{a}{r^2} + \ldots \infty\right)$, y = $\left(b - \frac{b}{r} + \frac{b}{r^2} - \ldots \infty\right)$ and z = $\left(c + \frac{c}{r^2} + \frac{c}{r^4} + \ldots \infty\right)$ then $\frac{xy}{z}$ = ?

    a) $\frac{c(a+b)}{ab}$ b) $c\sqrt{ab}$

    c) $\frac{ab}{c}$ d) $\frac{c}{ab}$

11. Find the sum of the geometric series (x + y) + (x^2 + xy + y^2) + (x^3 + x^2 y + xy^2 + y^3) + ... to n terms.

12. Two cars start together in the same direction from the same place. The first goes with uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increases the speed by $\frac{1}{2}$ km each succeeding hour. After how many hours will the second car overtake the first car if both cars go non-stop?

13. Which term of the sequence 12 + 8i, 11 + 6i, 10 + 4i, ... is (a) purely real (b) purely imaginary ?

14. Find a G.P. for which sum of the first two term is -4 and the fifth term is 4 times the third term.

15. The lengths of three unequal edges of a rectangular solid block are in GP. The volume of the block is 216 cm^3 and the total surface area is 252 cm^2. Find the length of the longest edge.

16. The angle between the two straight lines $6y^2 - xy - x^2 + 30y + 36 = 0$ is

    a) 30^o b) 50^o

    c) 45^o d) 60^o

17. The lines $x + (k - 1)y + 1 = 0$ and $2x + k^2y - 1 = 0$ are at right angles if

    a) $k > 1$ b) $k = 1$

    c) $k = - 1$ d) none of these

18. Determine the X-intercept a and the Y-intercept b of the lines $3x + 5y - 15 = 0$.

19. Find the distance of the point (2, 3) from the line $2x - 3y + 9 = 0$ measured along a line $x - y + 1 = 0$.

20. Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the positive direction of x-axis.
    [Hint: Use normal form, here $\omega = 30°$]

21. Find the image of the point (3, 8) with respect to the line $x + 3y = 7$ assuming the line to be a plane mirror.

22. If one diagonal of a square is along the line $8x – 15y = 0$ and one of its vertex is at (1, 2), then find the equation of sides of the square passing through this vertex.

23. A person standing at the junction (crossing) of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find equation of the path that he should follow.

24. Show that the tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2 - b^2}$.

25. If the circle $x^2 + y^2 + 2ax + 8y + 16 = 0$ touches x-axis, then the value of a is

    a) ± 16 b) ± 8

    c) ± 4 d) ± 1

26. If the equation of a circle is $\lambda x^2 + (2\lambda - 3) y^2 - 4x + 6y - 1 = 0$, then the coordinates of centre are

    a) $(\frac{2}{3}, -1)$ b) $(\frac{-2}{3}, 1)$

    c) $(\frac{4}{3}, -1)$ d) $(\frac{2}{3}, 1)$

27. Find the equation of a circle concentric with the circle $2x^2 + 2y^2 - 6x + 8y + 1 = 0$ and of double its area.

28. Find the equation of a circle of radius 5 which is touching another circle $x^2 + y^2 – 2x– 4y– 20 = 0$ at (5, 5).

29. Find the equation of the circle the end points of whose diameter are the centres of the circles $x^2 + y^2 + 6x - 14y - 1 = 0$ and $x^2 + y^2 - 4x + 10y - 2 = 0$.

30. If $f(x) = 1+ x + \frac{x^2}{2} + \cdots + \frac{x^{100}}{100}$, then f'(1) is equal to

    a) 0 b) $\frac{1}{100}$

    c) 100 d) does not exist

31. $\lim\limits_{x \to 1} (\cos[x])$ is equal to

    a) 0 b) cos 1

    c) does not exist d) 1

32. Show that $\lim_{x\to 4}\frac{|x-4|}{x-4}$ does not exist.

33. Differentiate $(\sin x + \cos x)^2$ w.r.t to x

34. If $y = \frac{1}{\sqrt{a^2-x^2}}$ find, $\frac{dy}{dx}$.

35. Evaluate the following limits: $\lim_{x\to 1}\frac{x-1}{\sqrt{x^2+3}-2}$.

36. Evaluate: $\lim_{x\to 0}\frac{a^x+b^x-2}{x}$.

37. Differentiate $x^2\cos x$ from the first principle.

38. Evaluate the following limits: $\lim_{x\to 4}\frac{2-\sqrt{x}}{4-x}$.

39. Differentiate each of the from first principles: $\sqrt{\tan x}$.

40. Evaluate: $\lim_{x\to\frac{\pi}{2}}\frac{\sqrt{2-\sin x}-1}{\left(\frac{\pi}{2}-x\right)^2}$.

41. Evaluate : $\operatorname{Lim}_{x\to 0}\frac{x(e^x-1)}{1-\cos x}$

42. Three numbers are chosen at random from 1 to 20. The probability that they are consecutive is

    a) $\frac{1}{60}$ b) $\frac{3}{190}$

    c) none of these d) $\frac{1}{57}$

43. Three squares of chess board are selected at random. The probability of getting 2 squares of one colour and other of a different colour is

    a) $\frac{3}{32}$ b) $\frac{16}{21}$

    c) $\frac{8}{21}$ d) $\frac{3}{8}$

44. In a single throw of three dice, find the probability of getting a total of 17 or 18.

45. The odds against a husband who is 45 years old, living till he is 70 are 7:5 and the odds against his wife who is now 36, living till she is 61 are 5 ; 3. Find the probability that none of them will be alive 25 years hence.

46. If 4-digit numbers greater than 5000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when the digits are repeated?

47. If A and B are mutually exclusive event, P(A) = 0.35 and P (B) = 0.45, find P ($A \cap B'$)

48. Let A, B, C be three events. If the probability of occurring exactly one event out of A and B is 1 - x, out of B and C is 1 - 2x, out of C and A is 1 - x, and that of occurring three events simultaneously is x^2, then prove that the probability that at least one out of A, B, C will occur is greater than $\frac{1}{2}$.

49. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that,

    i. you both enter the same section?

    ii. you both enter the different sections?

50. If the letters of the word ASSASSINATION are arranged at random. Find the probability that: Four S's come consecutively in the word.