ABHYAS PAPER 02 / JEE MAINS 2024

JEE main - Mathematics

Time Allowed: 1 hour **Maximum Marks: 100**

General Instructions:

- All questions are compulsory.
- There are 30 questions where the first 20 questions are MCQs and the next 10 questions are numerical.
- Section-A within each part is compulsory. Attempt any 5 questions from section-B within each part.
- You will get 4 marks for each correct response and 1 mark will be deducted for an incorrect answer.

MATHS (Section-A)

Attempt any 20 questions

1. Consider a function $f : \mathbb{N} \to \mathbb{R}$, satisfying f(1) + 2f(2) + 3f(3) +...+ xf(x) = x(x + 1) f(x); $x \geq 2$ with f(1) = 1. Then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to **[4]**

   a) 8000 b) 8100

   c) 8200 d) 8400

2. If α is the root of the equation x^2 - x + 2 = 0 then the value of $\frac{6(-\alpha^3+2\alpha^2-\alpha)}{\alpha^5-3\alpha^4+3\alpha^3-\alpha^2}$ is equal to: **[4]**

   a) 3 b) 12

   c) 6 d) 9

3. The number of triplets (x, y, z), where x, y, z are distinct non negative integers satisfying x + y + z = 15, is **[4]**

   a) 92 b) 114

   c) 80 d) 136

4. The sum of the series $\sum_{i=1}^{50} {}^{100}C_{50-i} \, {}^{50}C_i$ equals **[4]**

   a) $({}^{100}C_{50})^2$ b) ${}^{150}C_{50}$

   c) $({}^{100}C_{50})^2$ - ${}^{100}C_{50}$ d) ${}^{150}C_{50}$ - ${}^{100}C_{50}$

5. For any three positive real numbers a, b and c, if $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$, then **[4]**

   a) b, c and a are in AP b) b, c and a are in GP

   c) a, b and c are in GP d) a, b and c are in AP

6. If c ∈ [0, 1] then the minimum value of $\int_0^{4\pi/3} |\sin x - c| dx$ occurs when c is equal to : **[4]**

   a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{4}$

   c) $\frac{3}{4}$ d) $\frac{1}{2}$

7. The function f(x) = $\frac{\log(\pi+x)}{\log(e+x)}$ is **[4]**

a) Decreasing on (0, ∞)

b) Increasing on (0, ∞)

c) Decreasing on $(0, \frac{\pi}{e})$, increasing on $(\frac{\pi}{e}, \infty)$

d) Increasing on $(0, \frac{\pi}{e})$, decreasing on $(\frac{\pi}{e}, \infty)$

8. $\int \frac{\sin x+8\cos x}{4\sin x+6\cos x}dx$ = **[4]**

a) 2x + log |2 sin x + 3 cos x| + c

b) x + 2 log |2 sin x + 3 cos x| + c

c) x + $\frac{1}{2}$log |4 sin x + 6 cos x| + c

d) $\frac{1}{2}$log |4 sin x + 6 cos x| + c

9. Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is **[4]**

a) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$

b) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$

c) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$

d) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

10. If P is any point on the circle S_1 : $x^2 + y^2 = 144$ and Q is on the circle S_2: $x^2 + y^2 - 6x - 8y = 0$, then sum of maximum and minimum possible values of PQ, will be: **[4]**

a) 22

b) 44

c) 24

d) 20

11. If (x_1, y_1) and (x_2, y_2) are the end points of a latus rectum of the parabola $y^2 = 5x$, then $4x_1x_2 + y_1y_2$ = **[4]**

a) 0

b) $\frac{5}{4}$

c) 5

d) 25

12. Let y = y(x) be a solution curve of the differential equation, $(1 - x^2y^2)dx = ydx + xdy$. **[4]**
If the line x = 1 intersects the curve y = y(x) at y = 2 and the line x = 2 intersects the curve y = y(x) at y = α, then a value of α is

a) $\frac{1+3e^2}{2(3e^2-1)}$

b) $\frac{1-3e^2}{2(3e^2+1)}$

c) $\frac{3e^2}{2(3e^2+1)}$

d) $\frac{3e^2}{2(3e^2-1)}$

13. The equations of motion of a particle in parametric form are x = 2t + 1, y = 3t - 1, z = 4t + 1. The equation of path is: **[4]**

a) $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$

b) a straight line

c) a circle

d) Both circle and straight line

14. The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is **[4]**

a) 1

b) $3\sqrt{2}$

c) $2\sqrt{3}$

d) $\sqrt{6}$

15. An aeroplane flies around a square, the sides of which measure 100 miles each. The aeroplane covers at speed of 100 m/h on the first side, at 200 m/h on the second side. At 300 m/h the third side and 400 m/h on the fourth side. The average speed of the aeroplane around the square is **[4]**

a) 900 m/h

b) 200 m/h

c) 192 m/h

d) 195 m/h

16. Urn A contains 9 red balls and 11 white balls. Urn B contains 12 red balls and 3 white balls. One is to roll a single fair die. If the result is a one or a two, then one is to randomly select a ball from urn A. Otherwise one is to randomly select a ball from urn B. The probability of obtaining a red ball, is: **[4]**

a) $\frac{21}{35}$

b) $\frac{41}{60}$

c) $\frac{35}{60}$

d) $\frac{19}{60}$

17. Let f : (-1, 1) $\rightarrow$ R be such that f (cos 4θ) = $\frac{2}{2-\sec^2\theta}$ for $\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then the values of f$\left(\frac{1}{3}\right)$ are: **[4]**

a) $1 \pm \sqrt{\frac{1}{2}}$

b) $1 \pm \sqrt{\frac{2}{3}}$

c) $1 \pm \sqrt{\frac{3}{2}}$

d) $1 \pm \sqrt{\frac{1}{3}}$

18. The one which does not represent a hyperbola is: **[4]**

a) (x - 1)(y - 3) = 3

b) xy = 1

c) $x^2 - y^2 = 0$

d) $x^2 - y^2 = 5$

19. If A and B are two sets, then A $\cap$ (A $\cup$ B)′ is equal to **[4]**

a) ϕ

b) A

c) None of these

d) B

20. Let A be any 3 $\times$ 3 invertible matrices. Then which one of the following is not always true? **[4]**

a) adj (adj (A)) = |A| $\cdot$ (adj (A))$^{-1}$

b) adj (adj (A)) = |A| $\cdot$ A

c) adj (adj (A)) = $|A|^2 \cdot$ (adj (A))$^{-1}$

d) adj (A) = |A| $\cdot$ A^{-1}

MATHS (Section-B)

Attempt any 5 questions

21. The number of points, where the curve $y = x^5 - 20x^3 + 50x + 2$ crosses the x-axis, is _____. **[4]**

22. Let g(x) = f$\left[\frac{x}{f(x)}\right]$ where f(x) is a differentiable positive function on (0, ∞) such that f(1) = $f'(1)$. Determine $g'(1)$ **[4]**

23. Let $\sqrt{3}\hat{i} + \hat{j}$, $\hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect the origin O. If the distance of C from the bisectors of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is _____. **[4]**

24. If the area bounded by the curves f(x) = $[\cos^{-1}|\cos x\ I]^2$, g(x) = $[\cos^{-1} I\cos x\ |]$ and |x| = $\frac{\pi}{2}$ is $a\pi^3 + b\pi^2 + c$, then find the minimum value of (|a| + |b| + |c|). **[4]**

25. Let p be the perpendicular distance of point A (-2,3,1) from the line passing through the point B (-3, 5, 2) which makes equal angles with positive direction x, y and z axis. Then find the value of $30p^2$. **[4]**

26. The probability that an event A happens in one trial of an experiment, is 0.4. Three independent trials of the experiments are performed. The probability that event A happens at least once, is _____. **[4]**

27. Let a_1, a_2, a_3,....be terms of an arithmetic progression such that $\frac{a_1+a_2+\ldots\ldots+a_p}{a_1+a_2+\ldots\ldots+a_q} = \frac{p^2}{q^2}$, p $\neq$ q If $\frac{a_6}{a_{21}} = \frac{m}{n}$ (where m and n are in their lowest form), then find the value of (4m - n). **[4]**

28. In $\triangle$ABC, if sinA (sin A + cos B - sin B) + cos A (cos A + sin B + cos B) = 1 + sin C and a = 4, b = 3, then find the area of the $\triangle$ABC. **[4]**

29. The number of matrices of order 3 $\times$ 3, whose entries are either 0 or 1 and the sum of all the entries is a prime **[4]**

number, is _____.

30. Let f(x) and g(x) be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$, and $g(f(x)) = 4x^2 + 6x + 1$, then the value of f(2) + g(2) is _____. **[4]**