FUNCTION, LIMITS , C& D / JEE

JEE main - Mathematics

1. $\lim_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{\frac{1}{x^2}}$ is equal to:

a) $\frac{1}{e}$

b) $\frac{1}{e^2}$

c) e^2

d) e

2. $\lim_{n\to\infty}\left(\frac{\left[\frac{n}{5}\right]+\left[\frac{n}{7}\right]-\left[\frac{n}{20}\right]}{n}\right)$, n ∈ N, where [ ] represents the greatest integer function, is

a) $\frac{1}{140}$

b) 1

c) does not exist

d) $\frac{41}{140}$

3. If α and β are the roots of the equation $375x^2 - 25x - 2 = 0$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\alpha^r + \lim_{n\to\infty}\sum_{r=1}^{n}\beta^r$ is equal to

a) $\frac{29}{358}$

b) $\frac{7}{116}$

c) $\frac{21}{346}$

d) $\frac{1}{12}$

4. The number of points, where f(x) = max. (|tan x|, cos |x|) is non-differentiable in the interval $(-\pi, \pi)$, is

a) 3

b) 6

c) 2

d) 4

5. Let f : R → R and g : R → R be the functions such that

i. f' = g

ii. g' = -f

iii. f(0) = 0, g(0) = 1,

then $(f(x))^2 + (g(x))^2$ equals:

a) 2

b) 1

c) 3

d) 4

6. The derivative of $\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)$, with respect to $\frac{x}{2}$ where $\left(x \in \left(0, \frac{\pi}{2}\right)\right)$ is

a) 1

b) $\frac{2}{3}$

c) $\frac{1}{2}$

d) 2

7. The value of $\lim_{x\to\infty} x\left(\tan^{-1}\frac{x+2}{x+3} - \frac{\pi}{4}\right)$ is

a) $-\frac{1}{2}$

b) -2

c) 2

d) $\frac{1}{2}$

8. Let α and β be the roots of the equation, $ax^2 + bx + c = 0$ where $1 < \alpha < \beta$, then $\lim_{x\to\infty} \frac{|ax^2+bx+c|}{ax^2+bx+c} = 1$ when:

a) $a < 0$ and $\alpha < m < \beta$

b) $\frac{|a|}{a} = 1$ and $m > \alpha$

c) $a < 0$

d) $a > 0$

9. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1} x$ if $x \neq 0$, is:

a) $\frac{1}{2}$

b) -1

c) 1

d) $-\frac{1}{2}$

10. If $\lim_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right) = l$, then l belongs to

a) [-1, 0)

b) (-1, 1)

c) (-2, -1]

d) [1, 2)

11. Let $f(x) = (\log_2 x)^3 + x^3 \ \forall\ x > 0$, then the derivative of $f^{-1}(x)$ with respect to x at x = 9 is :

[Note : You can assume derivative of $\log_a x$ is $\frac{1}{x}$.]

a) $\frac{27}{2}$

b) $\frac{1}{27}$

c) 27

d) $\frac{2}{27}$

12. If $y = \frac{a+bx^{\frac{3}{2}}}{x^{\frac{5}{4}}}$ and y' = 0 at x = 5, then the ratio a : b is equal to:

a) 5 : 2

b) 3 : 5

c) 1 : 2

d) $\sqrt{5}$: 1

13. $\lim_{x\to 0}\frac{xe^x-\log(1+x)}{x^2}$ equals:

a) $-\frac{3}{2}$

b) $\frac{3}{2}$

c) $-\frac{2}{3}$

d) $\frac{2}{3}$

14. If $\lim_{t\to 0}$ k t cosec t = $\lim_{t\to 0}$ t cosec k t, then k equals

a) a number $\neq \pm 1$

b) -1

c) 1

d) 1 or -1

15. Consider a decreasing sequence (x_n) as $\tan^{-1} 2 = x_1 > x_2 > x_3 > ... > x_n > ... \infty$, of strictly positive terms such that $\sin(x_{n+1} - x_n) + 2^{-(n+1)} \sin x_n \sin x_{n+1} = 0$ for all $n \geq 1$. Then

a) $\cot x_n$ is not rational for all n

b) $\lim_{n\to\infty} x_n = \frac{\pi}{4}$

c) $\cot x_n = \frac{7}{8} \Rightarrow n > 3$

d) $\cot x_n > 1$ for all n

16. If $\lim_{x\to a}\frac{(x+2)^{\frac{3}{2}}-(a+2)^{\frac{3}{2}}}{x-a} = \frac{m}{n}(a+2)^{\frac{p}{q}}$, them (m + p) - (n + q) is

a) -1

b) 0

c) 2

d) 1

17. Let n be an integer and $x \in R$. A function f is defined as f(x) = cos (2 g (x)), where $g(x) = \lim_{n\to\infty} \tan^{-1}(nx)$, then

a) the function f is not continuous

b) $\lim_{x\to 0} f(x) = 1$

c) $\lim_{x \to \frac{\pi}{2}}$ f(x) = -1

d) the range of function f is [-1, 1]

18. Let a function f be continuous at x = a and a function g be defined as g(x) = (x - a) f(x). Then

a) g is not differentiable

b) g'(a) = f(a)

c) g'(a) = f(0)

d) g'(a) = 0

19. There exists a function f(x) satisfying f(0) = 1, f'(0) = -1, f(x) > 0, $\forall$ x and

a) -2 $\leq$ f''(x) -1, $\forall$ x

b) f''(x) < -2, $\forall$ x

c) f''(x) < 0, $\forall$ x

d) -1 < f''(x) < 0, $\forall$ x

20. For a function f, f' (x) = $\frac{x}{1+x^2}$ for all x. For all real a, b.

a) |f(a) + f(b)| $\leq \frac{1}{2}$ |a + b|

b) |f(a) + f(b)| $\geq \frac{1}{2}$ |a - b|

c) |f(a) - f(b)| $\leq \frac{1}{2}$ |a - b|

d) |f(a) + f(b)| $\geq \frac{1}{2}$ |a + b|

21. Consider the equation $x^4 + x^3 + x^2 - 1 = 0$ and the following statements:

I. The equation has at least one positive root.

II. The equation has a pair of complex roots.

Which of the statements is/are true?

a) I alone

b) II alone

c) I and II

d) neither of the two

22. Let f(x) = [x] sin π x, where [ ] represents the greatest integer function. The value of $\lim_{x \to k^-} \frac{f(x)-f(k)}{x-k}$, where k is an integer, is

a) $(-1)^k . k\pi$

b) $(-1)^k (k - 1)\pi$

c) $(-1)^{k-1} k\pi$

d) $(-1)^{k-1} (k - 1)\pi$

23. Let f : R $\to$ R be a function defined by f(x) = max {x, x^2}. Let S denote the set of all points in R, where f is not differentiable. Then:

a) {1}

b) ϕ (an empty set)

c) {0}

d) {0, 1}

24. Let y = f(x), where function f satisfies the relation f(x + y) = 2f(x) + x f(y) + $y\sqrt{f(x)}$ for all x, y $\in$ R and f'(0) = 0 then f'(4) equals:

a) -2

b) 16

c) 4

d) 2

25. Let f(x) = |x - 1| + |x -2|, I = $\int_0^3$ f(x) dx, M = the minimum value of f, N = f'(x) for s < -4 and C = the value of f''(4). Then the value of $\frac{M^2-N^2+IC}{2}$ is :

a) $\frac{-5}{2}$

b) $\frac{-3}{2}$

c) $\frac{3}{2}$

d) $\frac{5}{2}$

26. If $\lim_{x\to 0} \left(\left[\frac{\sin^{-1} x}{x}\right] + \left[\frac{2^2 \sin^{-1} 2x}{x}\right] + \left[\frac{3^2 \sin^{-1} 3x}{x}\right] + \ldots\ldots + \left[\frac{n^2 \sin^{-1} nx}{x}\right] \right)$ = 100, then the value of n, is :

[Note : [k] denotes the greatest integer less than or equal to k.]

a) 3

b) 5

c) 4

d) 2

27. If $y^2 + \log_e(\cos^2 x) = y$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then:

a) $|y'(0)| + |y''(0)|$ = 3

b) $|y''(0)|$ = 2

c) y''(0) = 0

d) $|y'(0)| + |y''(0)| =$ = 1

28. Let the function, f : [-7, 0] $\rightarrow$ R be continuous on [-7, 0] and differentiable on (-7, 0). If f(-7) = -3 and f'(x) d" 2, for all x $\in$ (-7, 0), then for all such functions f, f'(-1) + f(0) lies in the interval:

a) $(-\infty, 20]$

b) $[-\infty, 11]$

c) [-6, 20]

d) [-3, 11]

29. Let L = $\lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, a > 0. Given that L is finite, then

a) a = 2, L = $\frac{1}{32}$

b) a = 4, L = $\frac{1}{64}$

c) a = 2, L = $\frac{1}{64}$

d) a = 4, L = $\frac{1}{32}$

30. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1} x$, if $-1 \le x \le 1$, is:

a) $-\frac{1}{1+x^2}$

b) -2

c) $\frac{1}{1+x^2}$

d) 2

31. Let f : R $\rightarrow$ R be a differentiable function satisfying f'(3) + f'(2) = 0. Then $\lim_{x \to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to

a) 1

b) e^2

c) e

d) e^{-1}

32. Let $f(x) = \frac{\tan[x]\pi}{x^2+1}$. The number of points at which f is not continuous, is

a) 0

b) infinitely many

c) 2

d) 1

33. Let [t] denote the greatest integer $\le$ t. If for some $\lambda \in R$ - (0, 1), $\lim_{x \to 0}\left|\frac{1-x+|x|}{\lambda - x + [x]}\right|$ = L, then L is equal to:

a) 0

b) 1

c) $\frac{1}{2}$

d) 2

34. If $x^p y^q = (x+y)^{p+q}$, then $\frac{dy}{dx}$ =

a) $-\frac{y}{x}$

b) $-\frac{x}{y}$

c) $\frac{y}{x}$

d) $\frac{x}{y}$

35. If f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2, then the value of $\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x-a}$ is

a) -5

b) 5

c) None of these

d) $\frac{1}{5}$

36. $\lim_{n \to \infty} \frac{[1^2 x] + [2^2 x] + \ldots + [n^2 x]}{n^3}$, where [ ] represents the greatest integer function, is

a) $\frac{x}{3}$

b) $\frac{x}{12}$

c) $\frac{x}{6}$ d) $\frac{x}{4}$

37. For each t ∈ R, let [t] be the greatest integer less than or equal to t. Then, $\lim_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{15}{x}\right]\right)$

a) is equal to 15
b) is equal to 120
c) is equal to 0
d) does not exist (in R)

38. If $xe^{xy} = y + \sin^2 x$, then $\left(\frac{dy}{dx}\right)_{x=0}$ is:

a) 0
b) 2
c) -1
d) 1

39. If f(x) = max. {sin x, $\sin^{-1}$ (cos x)}, then:

a) f is non-differentiable at x = $\frac{n\pi}{2}, n \in I$
b) f is discontinuous at x = $\frac{n\pi}{2}, n \in I$
c) f is continuous every where but not differentiable
d) f is differentiable every where

40. $\lim_{n\to\infty} \frac{7^n}{n!}$ equals

a) 7
b) 0
c) 1
d) none of these

41. If f(x) = $\sin^{-1}$ {[3x + 2] - {3x + (x - {2x})}}, $x \in \left(0, \frac{\pi}{12}\right)$ and gof (x) = $x \forall x \in \left(0, \frac{\pi}{12}\right)$ then $g'\left(\frac{\pi}{6}\right)$ is equal to :
[Note : {y} and [y] denote fractional part function and greatest integer function respectively.]

a) $\frac{1}{8}$
b) $\frac{-\sqrt{3}}{4}$
c) $\frac{-1}{4}$
d) $\frac{\sqrt{3}}{8}$

42. $\lim_{x\to\infty} \frac{(x+1)^{10}+(x+2)^{10}+\ldots+(x+100)^{10}}{x^{10}+10^{10}}$ is equal to

a) 10
b) 1
c) 0
d) 100

43. If x = $\sqrt{2^{\text{cosec}^{-1} t}}$ and y = $\sqrt{2^{\sec^{-1} t}}$ $(|t| \geq 1)$, then $\frac{dy}{dx}$ is equal to:

a) $-\frac{y}{x}$
b) $-\frac{x}{y}$
c) $\frac{x}{y}$
d) $\frac{y}{x}$

44. Let a sequence of number is as follows

1
3 5
7 9 11
13 15 17 19
21 23 25 27 29
....
....

If t_n is the first term of n^{th} row then $\lim_{n\to\infty}\left(\sqrt{t_n} - n\right)$ is equal to

a) 1
b) $\frac{-1}{2}$
c) $\frac{1}{2}$
d) -1

45. The value of $\lim_{n\to\infty} \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\ldots\cos\left(\frac{x}{2^n}\right)$ is

a) $\frac{x}{\sin x}$
b) $\frac{\sin x}{x}$
c) none of these
d) 1

46. Let y = $f\left(\frac{2x-1}{2x+1}\right)$ and f'(x) = sin (x^2), then $\frac{dy}{dx}$ at x = -1 is:

a) 4 sin 9
b) sin 9
c) 9 sin 4
d) sin 4

47. Let f(x) = $\log_e$ (sin x), (0 < x < π) and g(x) = $\sin^{-1}(e^{-x})$, (x $\geq$ 0). If α is a positive real number such that a = (fog)' (α) and b = (fog) (α), then:

a) $a\alpha^2 + b\alpha$ - a = -2a^2
b) $a\alpha^2 - b\alpha - a$ = 1
c) $a\alpha^2 - b\alpha$ - a = 0
d) $a\alpha^2$ + $b\alpha$ + a = 0

48. If a function f given by $f(x) = \frac{(a^x-1)^3}{\sin(x\log a)\log(1+x^2\log a^2)}$ continuous at x = 0, then f(0) =

a) log a^{-1}
b) log a
c) log$\sqrt{a}$
d) 2 log a

49. If y = $\cot^{-1}(x^2)$, then $\frac{dy}{dx}$ is equal to:

a) $\frac{2x}{\sqrt{1+4x}}$
b) $-\frac{2x}{\sqrt{1+x^2}}$
c) $\frac{2x}{1+x^4}$
d) $-\frac{2x}{1+x^4}$

50. Consider, f(x) = min $\left\{x^3-1, -\frac{1}{4}(|x-2|+|x+2|), 7-x^3\right\}$ p and q denote number of points where f(x) is discontinuous and non-derivable in [-2, 3] respectively then p + q is:

a) 3
b) 1
c) 0
d) 4

51. Let x, y $\in$ R satisfying the equation $\cot^{-1}$ x + $\cot^{-1}$ y + $\cot^{-1}$ (xy) = $\frac{11\pi}{12}$, then the value of $\frac{dy}{dx}$ at x = 1 is :

a) $\left(\frac{1}{3}+\frac{1}{2\sqrt{3}}\right)$
b) $-\left(\frac{3+\sqrt{3}}{3}\right)$
c) $-\left(\frac{5+\sqrt{3}}{3}\right)$
d) $-\left(\frac{1}{3}+\frac{1}{2\sqrt{3}}\right)$

52. If f(x) = x^n, then the value of f(1) - $\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+\ldots+\frac{(-1)^n f^n(1)}{n!}$ is:

a) 2^{n-1}
b) 1
c) 2^n
d) 0

53. $\lim\limits_{x\to 0}\frac{15^x-5^x-3^x+1}{\tan^2 x}$ equals

a) log (5 - 3)
b) (log 5) (log 3)
c) log (15)
d) $\log\left(\frac{5}{3}\right)$

54. If y = $\frac{\sec x}{1+\frac{\operatorname{cosec} x}{1+\frac{\sec x}{1+\frac{\operatorname{cosec} x}{1+\ldots\text{ to }\infty}}}}$, then $\frac{dy}{dx}$ =

a) $\frac{\sec x\tan x+y\operatorname{cosec} x\cot x}{1+2y+\operatorname{cosec} x-\sec x}$
b) $\frac{(1+y)\sec x\tan x+y\operatorname{cosec} x\cot x}{1+2y+\operatorname{cosec} x-\sec x}$
c) $\frac{(1+y)\operatorname{cosec} x\cot x+y\sec x\tan x}{1+2y\operatorname{cosec} x+\sec x}$
d) $\frac{(1+y)\sec x\tan x-y\operatorname{cosec} x\cot x}{1+2y+\operatorname{cosec} x-\sec x}$

55. Let $x^k + y^k = a^k$, (a, k > 0) and $\frac{dy}{dx}+\left(\frac{y}{x}\right)^{\frac{1}{3}}$ = 0, then k is:

a) $\frac{4}{3}$
b) $\frac{2}{3}$

c) $\frac{1}{3}$

d) $\frac{3}{2}$

56. Let 'f' be derivable function $\forall x \in R$ such that $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}; \forall x, y \in R$. If f'(0) = -1 and f(0) = 1, then:

a) $f^{-1}(x) = -f(x)$

b) $f^{-1}(x) = 2f(x)$

c) $2f^{-1}(x) = f(x)$

d) $f^{-1}(x) = f(x)$

57. Let $f(x) = e^{ax} + e^{bx}$, $a \neq b$, for all $x \in R$ and f''(x) - 2f'(x) - 15f(x) = 0. Then |a - b| is:

a) 8

b) 2

c) 4

d) 6

58. Let A, B, P be the points the curve y = In x with their x coordinates as 1, 2 and t respectively $\lim_{t\to\infty} \cos\angle BAP$ is:

a) $\frac{1}{\sqrt{1+\ln^2 2}}$

b) $\frac{1}{1+\ln 2}$

c) In 2

d) $\sqrt{1+\ln^2 2}$

59. If f(1) = 3, f'(1) = 2, then $\frac{d}{dx}$(log f(e^x + 2x)) at x = 0 is:

a) $\frac{3}{2}$

b) $\frac{2}{3}$

c) 0

d) 2

60. $\lim_{x\to\frac{\pi}{2}} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x} =$

a) log a

b) log x

c) a

d) log 2

61. If (a + $\sqrt{2}b$ cos x)(a - $\sqrt{2}b$ cos y) = $a^2 - b^2$, where a > b > 0, then $\frac{dx}{dy}$ at $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$ is:

a) $\frac{2a+b}{2a-b}$

b) $\frac{a+b}{a-b}$

c) $\frac{a-b}{a+b}$

d) $\frac{a-2b}{a+2b}$

62. Let f(x) = sin (π {x}), $-\frac{\pi}{2} < x < \frac{\pi}{2}$, where { } represents the fractional part. The number of points at which f is not continuous, is

a) 0

b) 1

c) 2

d) 3

63. $\lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals

a) $2\sqrt{2}$

b) $\sqrt{2}$

c) 4

d) $4\sqrt{2}$

64. The value of $\lim_{n\to\infty} \frac{(4(n+1))!}{(n+1)^4(4n)!}$ is

a) 16

b) 256

c) 4

d) 0

65. If y = $e^{\sin^{-1}(t^2-1)}$ and x = $e^{\sec^{-1}\left(\frac{1}{t^2-1}\right)}$, then $\frac{dy}{dx}$ is equal to:

a) $\frac{y}{x}$

b) $-\frac{y}{x}$

c) $-\frac{x}{y}$

d) $\frac{x}{y}$

66. Let f(x) = $ax^3 + bx^2 + cx + 5$. If $|f(x)| \leq |e^x - e^2|$ for all $x \geq 0$ and if the maximum value of |12a + 4b + c| is l, the [l] is equal to :

[Note : [y] denotes greatest integer less than or equal to y.]

a) 7

b) 5

c) 6

d) 4

67. Let f : [-1,3] $\rightarrow$ R be defined as

$$f(x) = \begin{cases} |x| + [x], -1 \leq x < 1 \\ x + |x|, 1 \leq x < 2 \\ x + [x], 2 \leq x \leq 3 \end{cases},$$

Where, [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at

a) only one point

b) only three points

c) four or more points

d) only two points

68. $\lim_{x \to 0} \frac{\log \tan 2x}{\log \tan 3x} =$

a) $\frac{1}{e}$

b) e

c) 0

d) 1

69. Let $S_k(n) = 1^k + 2^k + ... + n^k$. Then $\lim_{n \to \infty} \frac{S_1(n)S_7(n) - (S_4(n))^2}{n^{10}}$ equals

a) $\frac{41}{400}$

b) $\frac{21}{80}$

c) $\frac{9}{400}$

d) $-\frac{11}{40}$

70. Let |x| < 1. If y = $\tan^{-1} \frac{2x}{1-x^2}$, then $\frac{dy}{dx}$ is equal to:

a) $\frac{1}{1+x^2}$

b) $\frac{2}{1-x^2}$

c) $\frac{1}{1-x^2}$

d) $\frac{2}{1+x^2}$

71. The value of $\lim_{x \to 0} \frac{(4^x - 1)^3}{\sin \frac{x^2}{4} \log(1+3x)}$ is

a) $\frac{3}{2}(\log_e 4)^2$

b) $\frac{4}{3}(\log_e 4)^3$

c) $\frac{3}{2}(\log_e 4)^4$

d) $\frac{4}{3}(\log_e 4)^2$

72. Let f(x) = $x^2 + \{x\}$, where { } represents the fractional part. Then

a) f is periodic function

b) f is not continuous at integral values of x

c) $\lim_{x \to 3}$ f(x) exists

d) $\lim_{x \to \frac{3}{2}} f(x) = 3$

73. If $\lim_{n \to \infty} \prod_{k=2}^{n} \frac{k^3 - 1}{k^3 + 1} = \frac{p}{q}$, then

a) 2q = 3p

b) p - q = 1

c) p + q = 7

d) pq = 3

74. Let f be a twice differentiable function such that f''(x) = -f(x) and f'(x) = g(x). A function h is defined as h(x) = $(f(x))^2 + (g(x))^2$. If h(5) = 4, then h(7) equals:

a) 14

b) 4

c) 8   d) 6

75. The function f given by $f(x) = [x]^2 - [x^2]$ is discontinuous at

a) all integers except 1   b) all integers except 0 and 1

c) all integers   d) all integers except 0