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JEE PAPER 02

JEE main - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 160**

General Instructions:

Think- Believe and then try to solve.

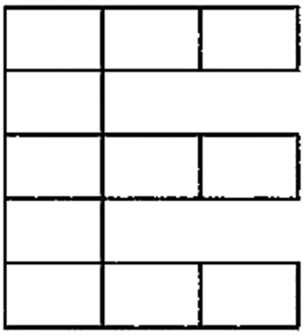
Section A

1. Given six line segments of length 2, 3, 4, 5, 6, 7 units the number of triangles that can be formed by these segments is **[4]**

a) ${}^6C_3$ - 4 b) ${}^6C_3$ - 5

c) ${}^6C_3$ - 6 d) ${}^6C_3$ - 7

2. The number of ways in which AABBCC can placed in squares of given figure so that no row remains empty are : **[4]**



a) 1620 b) 7290

c) 2430 d) 810

3. When expanded, the product (x + 2) (x + 3) (x + 4).... (x + 9)(x + 10) can be written as $a_9x^9 + a_8x^8 + ... + a_1x + a_0$. The value of $(a_1 + a_3 + a_5 + a_7 + a_9)$ is: **[4]**

a) 11! - 9! b) $\frac{(11!-9!)}{2}$

c) 27(9!) d) 11!

4. Sum of odd terms is A and sum of even terms is B in the expansion $(x + a)^n$, then **[4]**

a) $AB = \frac{1}{4}(x - a)^{2n} - (x + a)^{2n}$ b) $2AB = (x + a)^{2n} - (x - a)^{2n}$

c) none of these d) $4AB = (x + a)^{2n} - (x - a)^{2n}$

5. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is: **[4]**

a) 27 b) $\frac{21}{2}$

c) 16 d) 7

6. If f(x) = ||| x - 2| -5| -a| then number of integral values of 'a' for which f(x) has exactly 7 critical points, is **[4]**

a) 5 b) 9

c) 7 d) 4

7. The value for A, B and C respectively if $\lim\limits_{x \to 1} \frac{Ax^4+Bx^3+1}{(x-1)\sin \pi x}$ exists and is equal to C are: **[4]**

a) $3, -4, \frac{6}{\pi}$ b) $-4, 3, -\frac{6}{\pi}$

c) $3, -4, -\frac{6}{\pi}$ d) $-4, 3, \frac{6}{\pi}$

8. If f(x) = max. $\left(x^4, x^2, \frac{1}{81}\right) \forall x \in [0, \infty)$, then the sum of the square of reciprocal of all the values of x where f(x) is non-differentiable, is equal to: **[4]**

a) 1 b) 81

c) 82 d) $\frac{82}{81}$

9. $\int \frac{1}{\sin^3 x+\cos^3 x}$ dx equals **[4]**

a) $\frac{3}{2}$ log$\left|\frac{\sqrt{2}+\sin x-\cos x}{\sqrt{2}-\sin x+\cos x}\right|$ + c b) $\frac{1}{3\sqrt{2}}$ log $\left|\frac{\sqrt{2}+\sin x-\cos x}{\sqrt{2}-\sin x+\cos x}\right|$ + $\frac{2}{3}$ $\tan^{-1}$ (sin x - cos x) + c

c) $\frac{1}{3\sqrt{2}}$ $\tan^{-1}$ (sin x - cos x) + c d) $\frac{1}{3\sqrt{2}}$ log $\left|\frac{\sqrt{2}+\sin x-\cos x}{\sqrt{2}-\sin x+\cos x}\right|$ - $\frac{2}{3}$ $\tan^{-1}$ (sin x - cos x) + c

10. $\int \frac{\sin x-\cos x}{(\sin x+\cos x)\sqrt{\sin x \cos x+\sin^2 x \cos^2 x}}$ dx equals **[4]**

a) $\sec^{-1}$ (1 + sin 2x) + c b) $-\sec^{-1}$ (1 + sin 2x) + c

c) $-\sec^{-1}$ (1 + sin x) + c d) $\sec^{-1}$ (1 + sin x cos x) + c

11. The value of $\int\limits_{-\pi}^{\pi}$ (1 + cos x + cos 2x + ... + cos (2013x)) (1 + sin x + sin 2x + ... + (2013x)) dx , is : **[4]**

a) π b) 2π

c) 2013 π d) 0

12. Let y = y(x) be the solution of the differential equation, $(x^2 + 1)^2 \frac{dy}{dx} + 2x(x^2 + 1)y = 1$ such that y(0) = 0. If $\sqrt{a}y(1) = \frac{\pi}{32}$, then the value of a is **[4]**

a) $\frac{1}{4}$ b) 1

c) $\frac{1}{2}$ d) $\frac{1}{16}$

13. Let f be a differentiable function such that $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}, (x > 0)$ and f(1) $\neq$ 4. Then, $\lim\limits_{x \to 0^+} xf\left(\frac{1}{x}\right)$ **[4]**

a) exists and equals 0 b) exists and equals $\frac{4}{7}$

c) exists and equals 4 d) does not exist

14. a, b, c $\in$ R such that 3a + 2b + c = 1. The least value of $a^2 + b^2 + c^2$ is: **[4]**

a) $\frac{1}{14}$ b) $\frac{1}{7}$

c) $\frac{3}{14}$ d) $\frac{2}{7}$

15. Let θ be the angle between the lines **[4]**

$L_1 : \begin{bmatrix} x = 2t + 1 \\ y = -t + 1 \\ z = 3t + 1 \end{bmatrix}$ and $L_2 : \begin{bmatrix} x = 3s + 2 \\ y = 6s - 1 \\ z = 4 \end{bmatrix}$

where s, t $\in$ R . Then the value of $\int_0^\theta \frac{1}{1+\tan x}$ dx is equal to:

a) $\frac{\pi}{2}$

b) $\frac{\pi}{3}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{6}$

16. Two-line $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1}$ and $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates **[4]**

a) (2, 4, 7)

b) (- 2, 4, 7)

c) (2, - 4, - 7)

d) (2, - 4, 7)

17. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-collinear vectors in a plane such that $|\vec{a}| = 1$, $|\vec{b}| = 3$ and $|\vec{c}| = \sqrt{10}$. If $\vec{a} \times \vec{c} = \alpha$ and $\vec{b} \times \vec{c} = \beta$ where, $\alpha, \beta \in \left[\frac{\pi}{2}, \pi\right]$, then $\alpha + \beta$ equals: **[4]**

a) $\frac{3\pi}{2}$

b) $\frac{7\pi}{6}$

c) $\frac{7\pi}{4}$

d) $\frac{4\pi}{3}$

18. If p[th] q[th], r[th] terms of a G.P. are the positive numbers a, b, c respectively then angle between the vectors $\left(\log a^2\right) \hat{i} + \left(\log b^2\right) \hat{j} + \left(\log c^2\right) \hat{k}$ and $(q - r)\hat{i} + (r - p)\hat{j} + (p - q)\hat{k}$ is: **[4]**

a) $\frac{\pi}{2}$

b) $\cos^{-1}\left(\frac{pqr}{\sqrt{p^2+q^2+r^2}}\right)$

c) $\frac{\pi}{3}$

d) $\sin^{-1}\left(\frac{1}{\sqrt{a^2+b^2+c^2}}\right)$

19. If f (x) = $\pi\left(\frac{\sqrt{x+7}-4}{x-9}\right)$, then the range of function y = sin (2f(x)) is: **[4]**

a) $\left(0, \frac{1}{\sqrt{2}}\right]$

b) (0, 2]

c) (0, 1]

d) $\left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right]$

20. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2} + e^{-x^2}, g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then **[4]**

a) a = b = c

b) a = b and c $\neq$ b

c) a $\neq$ b and c $\neq$ b

d) a = c and a $\neq$ b

21. Let a_1, a_2, a_3, ... be an A.P. with a_6 = 2. Then the common difference of this A.P., which maximises the product a_1, a_4, a_5, is: **[4]**

a) $\frac{6}{5}$

b) $\frac{3}{2}$

c) $\frac{2}{3}$

d) $\frac{8}{5}$

22. The minimum value of $2^{\sin x} + 2^{\cos x}$ is: **[4]**

a) $2^{-1+\sqrt{2}}$

b) $2^{-1+\frac{1}{\sqrt{2}}}$

c) $2^{1\frac{1}{\sqrt{2}}}$

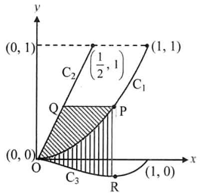
d) $2^{1-\sqrt{2}}$

[4]

23. Let $P(x) = a_0 + a_1 x^2 + a_2 x^4 + ... + a_n x^{2n}$, where $0 < a_1 < a_2 < ... < a_n$. Then P(x) has:

a) no point of relative optima
b) one point of relative maxima and one point of relative minima
c) exactly one point of relative minima
d) exactly one point of relative maxima

24. Let C_1 and C_2 be the curves given by $y - x^2$ and $y = 2x$, $0 \leq x \leq 1$, respectively. Let C_3 be a curve given by $y = f(x)$, $0 \leq x \leq 1$, $f(0) = 0$. From a point P on C_1, lines parallel to coordinate axes are drawn to meet C_2 and C_3 at Q and R respectively. If for every position P on C_1 the areas of shaded portions OPQ and OPR are equal, then the value of f(2) is: **[4]**



a) 8
b) 6
c) 4
d) 12

25. Let a function f satisfies the conditions $f(0) = 0$, $f'(0) = 1$ and $f''(x) = 2f'(x)$. Then, the area (in sq. units) bounded by the curve $y = f(x)$, $y = 0$ and $x - 1 = 0$ is equal to: **[4]**

a) $\frac{e^2-1}{2}$
b) $\frac{e^2-3}{4}$
c) $\frac{e^2}{2} - 1$
d) $e - 1$

26. Let $n \geq 2$ be a natural number and $0 < \theta < \frac{\pi}{2}$. Then, $\int \frac{(\sin^n \theta - \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$ is equal to (where C is a constant of integration) **[4]**

a) $\frac{n}{n^2-1} - \left(1 + \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$
b) $\frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$
c) $\frac{n}{n^2+1}\left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$
d) $\frac{n}{n^2-1}\left(1 - \frac{1}{\sin^{n+1} \theta}\right)^{\frac{n+1}{n}} + C$

Section B

27. Nine people sit a round table. The number of ways of selecting four of them such that they are not from adjacent seats, is **[4]**

28. The total number of 3-digit numbers, whose sum of digits is 10, is ________. **[4]**

29. Find the sum of all numbers between 200 and 400 which are divisible by 7. **[4]**

30. Evaluate $\int_{-\pi/2}^{\pi/2} (2\sin|x| + \cos x|) dx$. **[4]**

31. Let $f(k) = \int\limits_{-1}^{1} \frac{\sqrt{1-x^2}}{\sqrt{k+1} - x} dx$. If $\sum\limits_{k=0}^{99} f(k) = p\pi$, then find the value of p. **[4]**

32. Let y(x) be the solution of the differential equation $(x \log x) \frac{dy}{dx} + y = 2x \log x, (x \geq 1)$. Then, y(e) is equal to ________. **[4]**

33. If $4x + 4y - \lambda z = 0$ is the equation of the plane through the origin that contains the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$. Find the value of λ. **[4]**

34. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through (α, 7, 1) is , then α is equal to ________. **[4]**

35. Let $\vec{u}, \vec{v}$ and $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = \overrightarrow{0}$, If $|\vec{u}| = 3, |\vec{v}| = 4$ and $|\vec{w}| = 5$ then find **[4]**

$\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{w}+\vec{w}\cdot\vec{u}$.

36. Let A = {1, 2, 3, 4} and B = {a, b} be two sets. Write total number of onto functions from A to B. **[4]**

37. Find the least value of a such that the function f given by f(x) = x^2 + ax + 1 is strictly increasing on (1, 2). **[4]**

38. A polynomial function P(x) of degree 5 with leading coefficient one, increases in the interval $(-\infty$, 1) and (3, ∞) and decreases in the interval (1, 3). Given that P(0) = 4 and $P'(2)$ = 0. Find the value $P'(6)$. **[4]**

39. The number of distinct real roots of **[4]**

$x^4 - 4x^2 + 12x^2 + x - 1 = 0$ is ....

40. If $\int(x^{2010} + x^{804} + x^{402})\left(2x^{1608}+5x^{402}+10\right)^{\frac{1}{402}}$ dx = $\frac{1}{10a}\left(2x^{2010}+5x^{804}+10x^{402}\right)^{\frac{a}{402}}$, then find the value of a. **[4]**