I.S.C XL ABHYAS 01
Class 11 - Mathematics

1. A line passes through P(1, 2) such that its intercept between the axes is bisected at P. The equation of line is

   a) 2x + y = 4   b) x + y = 3

   c) x + 2y = 5   d) x- y + 1 = 0

2. For specifying a straight line, how many geometrical parameter should be known?

   a) 3   b) 2

   c) 1   d) 4

3. List all the proper subsets of {0, 1, 2, 3}.

4. If A and B are two sets. A has 20 elements, A $\cup$ B has 46 elements and A $\cap$ B has 15 elements, how many elements will B have?

5. Prove that: $\cos^2 x + \cos^2 (x + \frac{\pi}{3}) + \cos^2(x - \frac{\pi}{3}) = \frac{3}{2}$

6. Prove that: $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}$

7. If A + B = $\frac{\pi}{4}$, then prove that (1 + tan A)(1 + tan B) = 2

8. Prove that: $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2 \cos \theta$

9. If cube roots of unity are 1, ω and ω^2, then find the value of $(1 + \omega - \omega^2)^{32}$.

10. Evaluate : $(i^{77} + i^{70} + i^{87} + i^{414})^3$

11. If x + iy = $\frac{a-i}{a+i}$, then prove that ay + x + 1 = 0.

12. If ω is a complex cube root of unity and x = ω^2 - ω - 2, then find the value of $x^4 + 5x^3 + 9x^2$ - x - 21.

13. Let p, q be the roots of $3x^2 + 6x + 2 = 0$. Form an equation whose roots are $\frac{-p^2}{q}, \frac{-q^2}{p}$.

14. If **x** be real, find the maximum and minimum value of: $y = \frac{x+2}{2x^2+3x+6}$

15. If the roots of $ax^2 + x + b = 0$ are real and distinct, show that the roots of the equation $\frac{x^2+1}{x} = 4\sqrt{ab}$ are imaginary.

16. Find the range of x for the inequality $\frac{x^2-2x+5}{3x^2-2x-5} \geq \frac{1}{3}$.

17. Solve the linear inequality |x - 2| $\geq$ 6

18. A solution is to be kept between 68°F and 77°F. What is the range of temperature in degrees Celsius (°C) if the Celsius to Fahrenheit (F) conversion formula is given by F = $\frac{9}{5}C + 32$?

19. In how many ways can 5 different balls be distributed among 3 boxes?

20. The English alphabet has 5 vowels and 21 consonants. How many words with 2 different vowels and 2 different consonants can be formed from the alphabet?

21. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?

22. Ten IIT and 2 DCE students sit in a row. Find the number of ways in which 3 IIT students sit in between 2 DCE students.

23. Determine the number of 5 cards combinations out of deck of 52 cards if atleast one of the five cards has to be a king?

24. Using the digits 0, 1, 2, 3, 4, 5, 6 how many 4 digit even numbers can be made, no digit being repeated?

25. Find the coefficient of x^4 in $(1 - x)^2 (2 + x)^5$ using binomial theorem.

26. Find the 5th term from the end in the expansion of $\left(\frac{2}{x^2} - \frac{x^3}{4} \right)^8$.

27. Find n in the Binomial $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}} \right)^n$ if the ratio of 7th term from the beginning to the 7th term from the end is $\frac{1}{6}$.

28. Find the constant term in the expansion of $\left(x - \frac{1}{x} \right)^{12}$.

29. In the binomial expansion of $(\sqrt[3]{3} - \sqrt{2})^5$ find the term which does not contain irrational expression.

30. If $a^x = b^y = c^z$ and a, b, c are in G.P., then prove that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

31. If p times the pth term of an A.P. is q times the qth term, then show that its $(p + q)^{th}$ term is zero.

32. If $x = a + \frac{a}{r} + \frac{a}{r^2} + ...\infty$, $y = b - \frac{b}{r} + \frac{b}{r^2} + ...\infty$ and $z = c + \frac{c}{r^2} + \frac{c}{r^4} + ...\infty$ prove that $\frac{x \cdot y}{z} = \frac{ab}{c}$.

33. Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab}$ may be the geometric mean between n and b.

34. If p, q, r, are In G.P. and the equations $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in A.P.

35. If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2, b^2 are in A.P., then prove that $a^4 + b^4 = 0$.

36. Find the equations of the lines through the point (3, 2) which make an angle of 45° with the line x - 2 y = 3.

37. The line 2x - 3y = 4 is the perpendicular bisector of the line segment AB. If co-ordinates of A are (- 3, 1), find the coordinates of B.

38. Find the equations of circles which pass through the point (3, 6) and touch both the axes.

39. If y = 2x is a chord of the circle $x^2 + y^2 - 10x = 0$, then find the equation of a circle with this chord as diameter.

40. Find the equation of circle circumscribing the rectangle formed by the lines x = 6, x = -3, y = 3, and y = -1.

41. Evaluate $\lim\limits_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$

42. If $y = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$ find $\frac{dy}{dx}$

43. Evaluate : $\lim\limits_{x \to a} \frac{\sqrt{x} + \sqrt{a}}{x + a}$

44. If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, prove that $2x \cdot \frac{dy}{dx} + y = 2\sqrt{x}$.

45. Evaluate $\lim\limits_{x \to \sqrt{2}} \frac{x^4 - 4}{x^2 + 3\sqrt{2}x - 8}$

46. $\lim\limits_{x \to \frac{\pi}{6}} \frac{\sqrt{3}\sin x - \cos x}{x - \frac{\pi}{6}} = ?$

47. Using first principle find the derivative of $\frac{x+1}{x-1}$ with respect to x.

48. 1 boy and 2 girls are in a room A and 3 boys and 1 girl are in room B. Write the sample space for the experiment in which room is selected and then a person.

49. If E_1, E_2, E_3 are three mutually exclusive event and exhaustive events of an experiment such that $2P(E_1) = 3P(E_2) = P(E_3)$, then find $P(E_1)$.

50. In a lottery a person chooses six different natural numbers at random from 1 to 20 and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game?