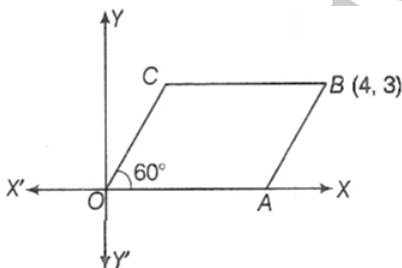


I.C.S.E ABHYAS 01

Class 10 - Mathematics

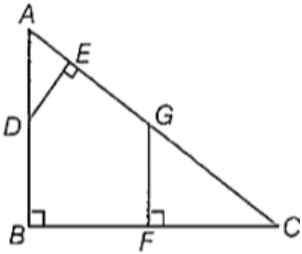
- The linear factors of the equation  $x^2 + kx + 1 = 0$  exists, if
  - Both  $k \geq 2$  and  $k \leq -2$
  - $k \geq 2$
  - $k = 2$
  - $k \leq -2$
- A manufacturer marks an article at ₹ 5000. He sells it to a wholesaler at a discount of 25% on the marked price and the wholesaler sells it to a retailer at a discount of 15% on the marked price. The retailer sells it to a consumer at the marked price. If all the sales are intra-state and the rate of GST is 12%, find
  - the amount inclusive of tax (under GST) that the wholesaler pays for the article.
  - the amount inclusive of tax (under GST) which the retailer pays for the article.
  - the amount of tax (under GST) that the wholesaler pays to the Central Government.
  - the amount of tax (under GST) that the retailer pays to the State Government.
- A shopkeeper buys an item whose printed price is ₹4000 from a wholesaler at a discount of 20% and sells it to a consumer at the printed price. If the sales are intra-state and the rate of GST is 12%, find
  - the price of the article inclusive GST in which the shopkeeper bought it.
  - the amount of tax (under GST) paid by the shopkeeper to the State Government.
  - the amount of tax (under GST) received by the Central Government.
  - how much the consumer pays the amount to the article?
- Ama ram has a Recurring Deposit Account in a bank for  $3\frac{1}{2}$  yr at 9.5% per annum. If he gets ₹58978, at the time of maturity, then find the monthly instalment.
- A man has a Recurring Deposit Account in a bank for  $3\frac{1}{2}$  yr. If the rate of interest is 12% per annum and the man gets ₹ 30618 on maturity, then find the value of monthly instalment.
- A man sells 100, ₹ 25 shares of a company paying 10% dividend, at ₹ 50 each and invests the proceeds in ₹ 5 shares of another company at ₹ 10 each. Find his change in income, if the second company pays a dividend of 6%.
- Mr. Sharma receives an annual income of ₹ 900 in buying 50 shares selling at ₹ 80. If the dividend declared is 20%, find the
  - Amount invested by Mr. Sharma.
  - Percentage return on his investment.
- Solve for x
$$4x^2 - 4a^2x + (a^4 - b^4) = 0$$
- For what value of k,  $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$  is a perfect square?
- B takes 16 days less than A to do a piece of work. If both, working together can do it in 15 days, then in how many days, will B alone complete the work?

11. An aeroplane flying with a wind of 30 km/h takes 40 min less to fly 3600 km than what it would to have taken to fly against the same wind. Find the aeroplane's speed of flying in still air.
12. Find the roots of equation  $\sqrt{2x-3} + \sqrt{3x-5} = 0$
13. If  $q$  is the mean proportional between  $p$  and  $r$ , then prove that  $\frac{p^3+q^3+r^3}{p^2q^2r^2} = \frac{1}{p^3} + \frac{1}{q^3} + \frac{1}{r^3}$ .
14. What least number must be added to each of the numbers 16, 7, 79 and 43, so that the resulting numbers are in proportion?
15. If  $ax = by = cz$ , then prove that  $\frac{x^2}{yz} + \frac{y^2}{zx} + \frac{z^2}{xy} = \frac{bc}{a^2} + \frac{ca}{b^2} + \frac{ab}{c^2}$
16. If  $p = \frac{4xy}{x+y}$ , then find the value of  $\frac{p+2x}{p-2x} + \frac{p+2y}{p-2y}$ .
17. If  $y = \frac{10ab}{a+b}$ , then find the value of  $\frac{y+5a}{y-5a} + \frac{y+5b}{y-5b}$ .
18. Use factor theorem to factorise  $6x^3 + 17x^2 + 4x - 12$  completely.
19. If  $(y - p)$  is a common factor of the polynomials,  $f(y) = y^2 + ay + b$  and  $g(y) = y^2 + my + n$  then show that  $p = \frac{n-b}{a-m}$ .
20. If a matrix has 24 elements, then what are the possible orders it can have?
21. Construct a matrix of order  $3 \times 2$ , whose elements are determined by  $a_{ij} = \frac{i+j}{2}$ .
22. Construct a  $3 \times 3$  matrix, whose elements are given by  $a_{ij} = \sin ix \operatorname{cosec} jx$ .
23. The ages of the students in a class are in AP, whose common difference is 4 months. If the youngest student is 8 yr old and the sum of the ages of all the students is 168 yr, then find the number of students in the class.
24. In a GP, if  $T_1 = 3$ ,  $T_n = 96$  and  $S_n = 189$ , then find  $n$ .
25. The sum of the first  $n$  terms of an AP is given by  $S_n = 2n^2 + 5n$ . Find the  $n$ th term of the AP.
26. Show that the ratio of the sum of the first  $n$  terms of a GP to the sum of terms from  $(n + 1)$ th to  $(2n)$ th term is  $\frac{1}{r^n}$ .
27. Find the reflection of the point  $(-1, 5)$  in the given point  $(2, 3)$ .
28. Find the value of  $p$ , if  $(p, -2)$ ,  $(-5, 6)$  and  $(1, 2)$  are collinear.
29. Find the ratio in which the line joining  $(-2, 5)$  and  $(-5, -6)$  is divided by the line  $y = -3$ . Hence find the point of intersection.
30. Find the equation of the line passing through the point of intersection of  $7x + 6y = 71$  and  $5x - 8y = -23$  and perpendicular to the line  $4x - 2y = 1$ .
31. Find the value of  $p$  if the lines,  $5x - 3y + 2 = 0$  and  $6x - py + 7 = 0$  are perpendicular to each other. Hence, find the equation of a line passing through  $(-2, -1)$  and parallel to  $6x - py + 7 = 0$ .
32. In the given figure, OABC is a parallelogram. Inclination of OC is  $60^\circ$  and the point B has coordinates  $(4, 3)$ . Find the equation of all sides of parallelogram OABC.

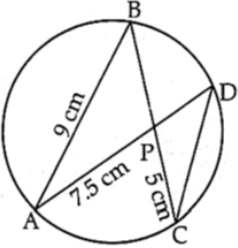


33. If the lines  $3(\alpha - 1)y - 6x = 2$  and  $4y - 8x + 10 = 0$  are parallel, then find the value of  $\alpha$ .

34. In the given figure, if  $AB \perp BC$ ,  $DE \perp AC$  and  $GF \perp BC$ , then prove that  $\triangle ADE \sim \triangle GCF$ .

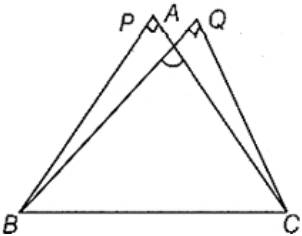


35. In the given figure  $AB = 9$  cm,  $PA = 7.5$  cm and  $PC = 5$  cm. Chords AD and BC intersect at P.



- i. Prove that  $\triangle PAB \sim \triangle PCD$
- ii. Find the length of CD.
- iii. Find area of  $\triangle PAB$  : area of  $\triangle PCD$

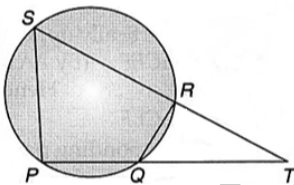
36. In  $\triangle ABC$ ,  $\angle A$  is obtuse,  $PB \perp PC$  and  $QC \perp QB$ . Prove that  $AB \times AQ = AC \times AP$ .



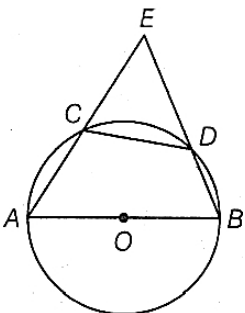
37. Prove that the area of the equilateral triangle described on the side of an isosceles right angled triangle is half the area of the equilateral triangle described on its hypotenuse.

38. In the given figure PQRS is a cyclic quadrilateral PQ and SR produced meet at T.

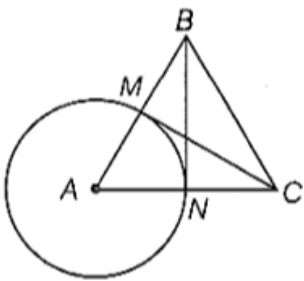
- i. Prove  $\triangle TPS \sim \triangle TRQ$ .
- ii. Find SP if  $TP = 18$  cm,  $RQ = 4$  cm and  $TR = 6$  cm.
- iii. Find area of quadrilateral PQRS if area of  $\triangle PTS = 27$  cm<sup>2</sup>.



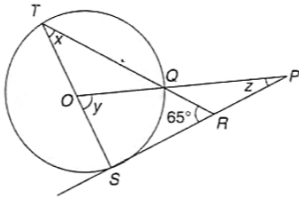
39. In the adjoining figure, AB is a diameter of a circle with centre O and CD is a chord equal to the radius of the circle. AC and BD when extended intersect at E. Prove that  $\angle AEB = 60^\circ$ .



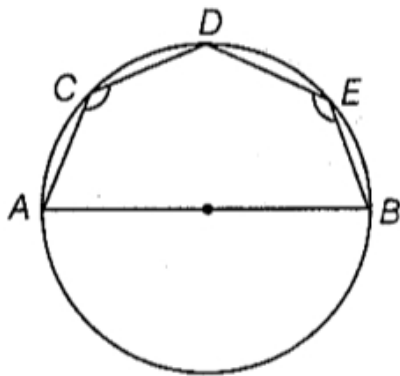
40. ABC is an equilateral triangle. A circle is drawn with centre A, so that it cuts AB and AC at points M and N, respectively. Prove that  $BN = CM$ .



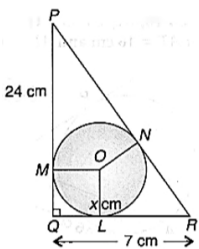
41. In the figure given below, O is the centre of the circle and SP is a tangent. If  $\angle SRT = 65^\circ$ , find the values of x, y and z.



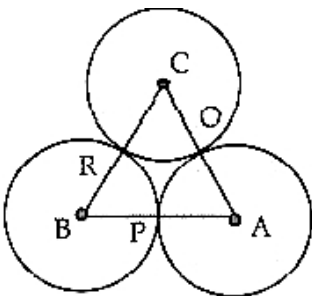
42. In the given figure, AB is the diameter of the circle and C, D, E are any three points on the semi-circle. Find the value of  $\angle ACD + \angle BED$ .



43. In  $\triangle PQR$ ,  $PQ = 24$  cm,  $QR = 7$  cm and  $\angle PQR = 90^\circ$ . Find the radius of the inscribed circle.



44. ABC is a triangle with  $AB = 10$  cm,  $BC = 8$  cm and  $AC = 6$  cm (not darers to scale). Three circles are drawn touching each other with vertices as their centres. Find the radii of three circles.



45. If a, b and c are the sides of a right triangle, where c is hypotenuse, then prove that the radius r of the circle which touches the sides of the triangle, is given by  $r = \frac{a+b-c}{2}$ .

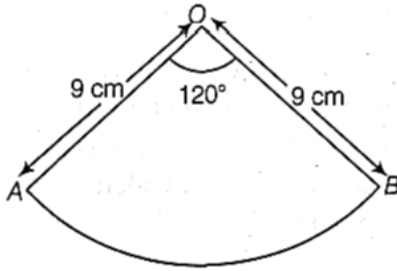
46. A cone and a cylinder are having equal base radius. Find the ratio of the heights of cone and cylinder, if their volumes are equal.

47. A roadroller (in the shape of a cylinder) has a diameter 0.7 m and its width is 1.2 m. Find the least number of revolutions that the roller must make in order to level a playground of size 120 m by 44 m.

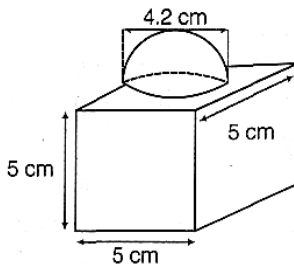
48. The height of a cone is 40 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume be  $\frac{1}{64}$  of the volume of the given cone, at what height above the base is the section cut?

49. The volumes of two spheres are in the ratio 64 : 27. If the sum of their radii is 21 cm, then find their radii.

50. In the given figure, A sector of a circle of radius 9 cm and central angle of  $120^\circ$ . It is rolled up so that the two bounding radii are joined together to form a cone. Find



- i. the slant height of the cone.
  - ii. the radius of the base of the cone.
  - iii. the volume of the cone.
  - iv. the total surface area of the cone.
51. The decorative block as shown in the figure is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge 5 cm and the hemisphere fixed on the top has a diameter of 4.2 cm, then find the total surface area of the block and find the total area to be painted.



52. Prove that:  $\frac{\sin^4 \theta + \cos^4 \theta}{1 - 2 \sin^2 \theta \cos^2 \theta} = 1$

53. Prove that:  $\frac{1}{\sec x - \tan x} + \frac{1}{\sec x + \tan x} = \frac{2}{\cos x}$

54. Prove that

i.  $\sec A (1 - \sin A) (\sec A + \tan A) = 1$

ii.  $\frac{\cos A}{1 + \sin A} + \tan A = \sec A$

iii.  $\frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta} = \cos \theta + \sin \theta$

iv.  $\frac{\tan^2 \theta}{(\sec \theta - 1)^2} = \frac{1 + \cos \theta}{1 - \cos \theta}$

55. Prove that:  $(\tan \theta + 2) (2 \tan \theta + 1) = 5 \tan \theta + 2 \sec^2 \theta$

56. If  $\tan \theta = 1$  and  $\sin \phi = \frac{1}{\sqrt{2}}$ , then find the value of  $\cos(\theta + \phi)$ , where  $\theta$  and  $\phi$  are both acute angles.

57. Prove that:  $2 \sec^2 \theta - \sec^4 \theta - 2 \operatorname{cosec}^2 \theta + \operatorname{cosec}^4 \theta = \cot^4 \theta - \tan^4 \theta$

58. Prove that:  $(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A \sec^2 B$

59. If  $1 + \cos^2 \theta = 3 \sin \theta \cos \theta$ , then prove that  $\tan \theta = 1$  or 2.

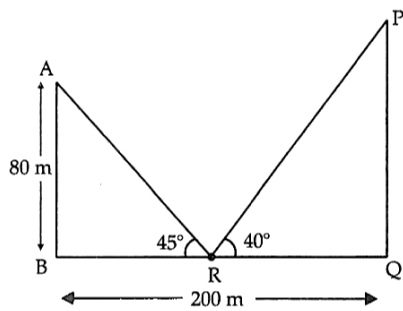
60. If  $x = r \sin A \cos B$ ,  $y = r \sin A \sin B$  and  $z = r \cos A$ , then show that  $x^2 + y^2 + z^2 = r^2$ .

61. If  $\sqrt{3} \cot^2 \theta - 4 \cot \theta + \sqrt{3} = 0$ , then find the value of  $\tan^2 \theta + \cot^2 \theta$ .

62. A person, standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank is  $60^\circ$ . When he moves 50 m away from the bank, he finds that the angle of elevation to be  $30^\circ$ . Find the height of the tree and the breadth of the river.

63. Two poles AB and PQ are standing opposite each other on either side of a road 200 m wide. From a point R between them on the road, the angle of elevation of the top of the poles AB and PQ are  $45^\circ$  and  $40^\circ$  respectively. If height of AB

= 80 m, find the height of PQ correct to the nearest meter. [ $\because \tan 40^\circ = 0.8391$ ]



64. At the foot of a mountain, the angle of elevation of its summit is  $45^\circ$ . After ascending 1000 m towards the mountain up a slope of  $30^\circ$  inclination, the elevation is found to be  $60^\circ$ . Find the height of the mountain.
65. A fire at a building B is reported on telephone to two fire stations  $F_1$  and  $F_2$  10 km apart from each other.  $F_1$  observes that the fire is at an angle of  $50^\circ$  from it and  $F_2$  observes that it is at an angle of  $45^\circ$  from it. Which station should send its team and how much distance it has to travel?
66. If the median of the following distribution is 58 and the sum of all the frequencies is 140. Find the values of x and y.

Variable	15-25	25-35	35-45	45-55	55-65	65-75	75-85	85-95
Frequency	8	10	x	25	40	y	15	7

67. If median of the number of patients attending a hospital is 36, then find the missing frequencies  $f_1$  and  $f_2$  in the following frequency distribution, when it is given that total number of day is 100.

Number of patients	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of days	5	12	$f_1$	$f_2$	15	11	14

68. For the following distribution, draw a histogram.

Weight (in kg)	44-47	48-51	52-55	56-59	60-63	64-67
Number of shops	20	28	36	16	8	4

From the histogram estimate the mode.

69. Two friends were born in the year 2000. What is the probability that they have the same birthday?
70. A die has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and the total score is recorded.
- How many different scores are possible.
  - What is the probability of getting a total of 7?