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CALCULUS JEE PAPER 01

JEE main - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 100**

General Instructions:

Think Believe and solve

1. If $\lim\limits_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}$ = 1, then b - 3a equals: **[4]**

   a) -6 b) 1

   c) 2 d) 6

2. If f(x) = $\lim\limits_{n\to\infty}\frac{x^{3n}\sin x+\cos x}{x^{3n}+2}$, then $f\left(\frac{\pi}{6}\right)+f\left(\frac{\pi}{3}\right)$ is **[4]**

   a) $\frac{\sqrt{3}}{2}$ b) $\frac{2\sqrt{3}+1}{4}$

   c) $\frac{3\sqrt{3}}{4}$ d) $2\sqrt{3}$

3. Let f(x) = $\begin{cases}(x-1)^{\frac{1}{2-x}}, & x>1, x\neq 2\\ k, & x=2\end{cases}$ **[4]**

   The value of k for which f is continuous at x = 2 is

   a) 1 b) e^{-2}

   c) e^{-1} d) e

4. A function f : R $\to$ R satisfies **[4]**

   i. f(x) + f(y) $= f\left(\frac{x+y}{1-xy}\right)$ for all x, y such that xy $\neq$ 1

   ii. $\lim\limits_{x\to 0}\frac{f(x)}{x}=2.$

   Then f'(1)

   a) is 0 b) is -1

   c) is 1 d) does not exist

5. Let f(x) = $e^{\left\{e^{|x|}\operatorname{sgn} x\right\}}$, g(x) = $e^{\left[e^{|x|}\operatorname{sgn} x\right]}$, x $\in$ R, where **[4]**

   sgn (x) = -1, x < 0

   = 0, x = 0

   = 1, x > 0

   and, { } and [ ] represent fractional part and greatest integer function respectively. If h(x) = log f (x) + log g(x), then which of the following statement is correct?

   a) $\lim\limits_{x\to 0^+}$ h(x) = h(0) b) $\lim\limits_{x\to 0^+}\frac{h(x)-1}{x}$ = 1

   c) $\lim\limits_{x\to 0^-}$ h(x) = 1 d) $\lim\limits_{x\to 0^+}$ h(x) does not exist

6. Let α(a) and β(a) be the roots of the equation $(\sqrt[3]{1+a}-1)x^2-(\sqrt{1+a}-1)x+(\sqrt[6]{1+a}-1)=0$, where a > -1. Then, $\lim\limits_{a\to0^+}\alpha(a)$ and $\lim\limits_{a\to0^+}\beta(a)$ are **[4]**

a) $-\frac{9}{2}$ and 3
b) $-\frac{7}{2}$ and 2
c) $-\frac{1}{2}$ and -1
d) $-\frac{5}{2}$ and 1

7. If the value of the integral $\int\limits_0^{\frac{1}{2}}\frac{x^2}{(1-x^2)^{\frac{3}{2}}}$ dx is $\frac{k}{6}$, then k is equal to: **[4]**

a) $3\sqrt{2}+\pi$
b) $2\sqrt{2}-\pi$
c) $3\sqrt{2}-\pi$
d) $2\sqrt{2}+\pi$

8. Infinite rectangles each of width 1 unit and height $\left(\frac{1}{n}-\frac{1}{n+1}\right)(n\in N)$ are constructed such that ends of exactly one diagonal of every rectangle lies along the curve y = $\frac{1}{x}$. The sum of areas of all such rectangles, is : **[4]**

a) 1
b) $\frac{1}{2}$
c) $\frac{2}{3}$
d) $\frac{3}{4}$

9. Let f : $\left[0,\frac{\pi}{2}\right]\rightarrow$ R be continuous and satisfy f'(x) = $\frac{1}{1+\cos x}$ for all $x\in\left(0,\frac{\pi}{2}\right)$. If f(0 = 3 then f $\left(\frac{\pi}{2}\right)$ has the value equal to: **[4]**

a) none of these
b) 2
c) $\frac{13}{4}$
d) 4

10. The value of $\lim\limits_{n\to\infty}\frac{(1^1+2^2+\ldots+n^2)(1^3+2^3+\ldots+n^3)(1^4+2^4+\ldots+n^4)}{(1^5+2^5+\ldots.+n^5)^2}$ is equal to: **[4]**

a) $\frac{1}{5}$
b) $\frac{2}{5}$
c) $\frac{4}{5}$
d) $\frac{3}{5}$

11. If f(x) is differentiable and $\int_0^{t^2}xf(x)dx=\frac{2}{5}t^5$ then $f\left(\frac{4}{25}\right)$ equals **[4]**

a) $\frac{2}{5}$
b) $-\frac{5}{2}$
c) $\frac{5}{2}$
d) 1

12. The value of $\int\limits_{-1}^{1}\min.$(|x|, |x - 1|, |x + 1|)dx is **[4]**

a) 2
b) 0
c) -2
d) $\frac{1}{2}$

13. If y = y(x) satisfies the differential equation $8\sqrt{x}(\sqrt{9+\sqrt{x}})dy=(\sqrt{4+\sqrt{9+\sqrt{x}}})^{-1}dx, x>0$ and $y(0)=\sqrt{7}$, then y(256) = **[4]**

a) 16
b) 9
c) 80
d) 3

14. If the differential equation representing the family of all circles touching x-axis at the origin is $(x^2 - y^2)\frac{dy}{dx}$ = g(x)y, then g(x) equals: **[4]**

a) $2x^2$
b) 2x
c) $\frac{1}{2}x$
d) $\frac{1}{2}x^2$

15. Let y = y(x) be the solution curve of the differential, $\left(y^2-x\right)\frac{dy}{dx}$ = 1, satisfying y(0) = 1. This curve. This **[4]**

curve intersects the x-axis at a point whose abscissa is:

a) -e
b) 2 - e
c) 2
d) 2 + e

16. The solution of the differential equation, $\frac{dy}{dx} = (x-y)^2$, when y(1) = 1, is **[4]**

a) $-\log_e\left|\frac{1+x-y}{1-x+y}\right| = x+y-2$
b) $\log_e\left|\frac{2-x}{2-y}\right| = x-y$
c) $-\log_e\left|\frac{1-x+y}{1+x-y}\right| = 2(x-1)$
d) $\log_e\left|\frac{2-y}{2-x}\right| = 2(y-1)$

17. The range of function f (x) = sgn (sin x) + sgn (cos x) + sgn (tan x) + sgn (cot x), $x \neq \frac{n\pi}{2}(n \in \mathrm{I})$ is : **[4]**
[Note: sgn k denotes signum function of k.]

a) {-2, 0, 4}
b) {-4, -2, 0, 4}
c) {-2, 4}
d) {0, 2, 4}

18. Let $f'(x) = \frac{x}{(1+x^n)^{1/n}}$ for n $\geq$ 2 and $g(x) = \underbrace{(\text{ fofo...of })}_{f \text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2}$ g(x)dx equals. **[4]**

a) $\frac{1}{n(n+1)}(1+nx^n)^{1+\frac{1}{n}} + K$
b) $\frac{1}{n(n-1)}(1+nx^n)^{1-\frac{1}{n}} + K$
c) $\frac{1}{n+1}(1+nx^n)^{1+\frac{1}{n}} + K$
d) $\frac{1}{n-1}(1+nx^n)^{1-\frac{1}{n}} + K$

19. If f(x) = {x} +$\left\{x + \left[\frac{x}{1+x^2}\right]\right\} + \left\{x + \left[\frac{x}{1+2x^2}\right]\right\} + \ldots\ldots + \left\{x + \left[\frac{x}{1+99x^2}\right]\right\}$ then value of $[f(\sqrt{3})]$ is: **[4]**
Note : [k] and {k} denote greatest integer and fractional part functions of k respectively.

a) 17
b) 73
c) 5050
d) 4950

20. The domain of the function f defined as f(x) = $\log_{10}[1 - \log_{10}(x^2 - 5x + 16)]$ is **[4]**

a) (2, 3]
b) [2, 3]
c) (2, 3)
d) [2, 3)

21. Let f be a differentiable function satisfying the functional rule f(xy) = f(x) + f(y) + $\frac{x+y-1}{xy} \forall x$, y > 0 and f'(1) = 2. **[4]**
Find the value of [f(e^{100})].
Note: [k] denotes the greatest integer less than or equal to k.

22. Let f : (0, 1) $\rightarrow$ (0, 1) be a differentiable function such that f'(x) $\neq$ 0 for all x $\in$ (0, 1) and f$\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$. If f(x) = $\lim\limits_{t \to x} \frac{\int_0^t \sqrt{1-f^2(s)}ds - \int_0^x \sqrt{1-f^2(s)}ds}{f(t)-f(x)}$, then the value of f$\left(\frac{1}{4}\right)$ equals $\frac{\sqrt{m}}{4}$ where m $\in$ N. Find the value of m. **[4]**

23. If f : R $\rightarrow$ R is a continuous and differentiable function such that, $\int_{-1}^{x} f(t)$ + f''' (3)$\int_x^0 dt$ = $\int_1^x t^3$dt - f'(1)$\int_x^2 t^2$ dt + f''(2)$\int_3^x t$ dt, then find the value of f'(4). **[4]**

24. Let y'(x) + y(x) g'(x) = g(x) g'(x), y(0) = 0, x $\in$ R, where f'(x) denotes $\frac{df(x)}{dx}$ and g(x) is a given non-constant differentiable function on R with g(0) = g(2) = 0. Then, the value of y(2) is **[4]**

25. Let A = {1, 2, 3, 4} and B = {a, b} be two sets. Write total number of onto functions from A to B. **[4]**