VECTOR AND 3-D JEE BY HITESH SIR

JEE main - Mathematics

Section A

1. The equation of a plane containing the line of intersection of the planes 2x - y - 4 = 0 and y + 2z - 4 = 0 and passing through the point (1, 1, 0) is **[4]**

   a) x + 3y + 2 = 4   b) x - 3y - 2z = - 2

   c) 2x - z = 2   d) x - y - z = 0

2. If the points (1, 1, k) and (-3, 0, 1) be equidistant from the plane 3x + 4y - 12z + 13 = 0, then k = **[4]**

   a) 4   b) 1

   c) 2   d) 0

3. Given that A (1, -1, 1), B (3, 2, 2), C (1, 2, 3) and D (3, 0, 1) are four points. Consider the equation of a plane which is parallel to the plane 3x - 2y + 4z = 1 and passing through (2, -1, 1). Which of the following line segments is intersected by the plane? **[4]**

   a) None of these   b) AC

   c) BD   d) BC

4. The distance of the point (1, -2, 3) from the plane x - y + z = 5 measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is: **[4]**

   a) $\frac{7}{5}$ b) 1

   c) 7   d) $\frac{1}{7}$

5. Let $\theta_1, \theta_2, \theta_3$ be the angles made by a line with the coordinate planes. Then $\sum\limits_{i=1}^{3} \cos^2 \theta_i$ equals: **[4]**

   a) 3   b) 1

   c) 0   d) 2

6. The length of the perpendicular from the point (2, -1, 4) on the straight line $\frac{x+3}{10} = \frac{y-2}{-7} = \frac{z}{1}$ is **[4]**

   a) greater than 3 but less than 4   b) greater than 4

   c) greater than 2 but less than 3   d) less than 2

7. A plane passes through (1, - 2, 1)and is perpendicular to two planes 2x - 2y + z = 0 and x - y + 2z = 4, then the distance of the plane from the point (1, 2, 2) is **[4]**

   a) $\sqrt{2}$ b) 1

   c) 0   d) $2\sqrt{2}$

8. If the point (2, α, β) lies on the plane which passes through the points (3, 4, 2) and (7, 0, 6) and is perpendicular to the plane 2x - 5y = 15, then 2α - 3β is equal to **[4]**

a) 5
b) 17
c) 12
d) 7

9. If A = (2, 3, 4), B = (4, 7, 14) and P = (λ, 0, 0) are three points such that the distance of point P from the plane bisecting line AB at right angles is 11 units, then λ, satisfies the equation: **[4]**

a) $x^2 - 58x + 64 = 0$
b) $x^2 - 116x - 266 = 0$
c) $x^2 + 116x - 266 = 0$
d) $x^2 + 58x - 64 = 0$

10. The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes 2x + y - 2z = 5 and 3x - 6y - 2z = 7 is **[4]**

a) 14x + 2y - 15z = 1
b) -14x + 2y + 15z = 3
c) 14x + 2y + 15z = 31
d) 14x - 2y + 15z = 27

11. If the lines $\frac{x-1}{2} = \frac{y-k}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{k}$ = z intersect, then the value of k is: **[4]**

a) 1
b) 4
c) -2
d) 2

12. The mirror image of the point (1, 2, 3) in a plane is $\left(-\frac{7}{3}, -\frac{4}{3}, -\frac{1}{3}\right)$. Which of the following points lies on this plane? **[4]**

a) (-1, -1, -1)
b) (1, -1, 1)
c) (-1, -1, 1)
d) (1, 1, 1)

13. Two-line $\frac{x-3}{1} = \frac{y+1}{3} = \frac{z-6}{-1}$ and $\frac{x+5}{7} = \frac{y-2}{-6} = \frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates **[4]**

a) (2, 4, 7)
b) (- 2, 4, 7)
c) (2, - 4, - 7)
d) (2, - 4, 7)

14. Perpendicular are drawn from points on the line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3}$ to the plane x + y + z = 3. The feet of perpendiculars lie on the line **[4]**

a) $\frac{x}{4} = \frac{y-1}{3} = \frac{z-2}{-7}$
b) $\frac{x}{5} = \frac{y-1}{8} = \frac{z-2}{-13}$
c) $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{-5}$
d) $\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$

15. The equation of the plane, which passes through y-axis and is perpendicular to the line $\frac{x-5}{\cos\theta} = \frac{y+1}{0} = \frac{z-1}{\sin\theta}$, is: **[4]**

a) x sin θ + z cos θ = 0
b) x cos θ + z sin θ = 0
c) x cos θ +y sin θ = 0
d) y cos θ + z sin θ = 0

16. Let P be a plane passing through the points (2, 1, 0), (4, 1, 1) and (5, 0, 1) and R be any point (2, 1, 6). Then the image of R in the plane P is: **[4]**

a) (6, 5, -2)
b) (4, 3, 2)
c) (3, 4, -2)
d) (6, 5, 2)

17. If d_1, d_2, d_3 denote the distances of the plane 2x - 3y + 4z + 2 = 0 from the planes 2x - 3y + 4z + 6 = 0, 4x - 6y + 8z + 3 = 0 and 2x - 3y + 4z - 6 = 0 respectively, then: **[4]**

a) $d_1 + 16d_2 = 0$
b) $d_1 - 2d_2 + 3d_3 = \sqrt{29}$

c) $8d_2 = d_1$ d) $d_1 + 8d_2 + d_3 = 0$

18. The shortest distance between the lines $\frac{x-1}{0} = \frac{y+1}{-1} = \frac{z}{1}$ and x + y + z + 1 = 0, 2x - y + z + 3 = 0 is: **[4]**

a) $\frac{1}{\sqrt{3}}$ b) $\frac{1}{2}$

c) $\frac{1}{\sqrt{2}}$ d) 1

19. If the plane 2x - y + 2z + 3 = 0 has the distances $\frac{1}{3}$ and $\frac{2}{3}$ units from the planes 4x - 2 y + 4z + λ = 0 and 2x - y + 2z + μ = 0, respectively, then the maximum value of λ + μ is equal to **[4]**

a) 15 b) 5

c) 13 d) 9

20. The distance of the point having position vector $-\hat{i} + 2\hat{j} + 6\hat{k}$ from the straight line passing through the point (2,3, - 4) and parallel to the vector $6\hat{i} + 3\hat{j} - 4\hat{k}$ is **[4]**

a) 6 b) $2\sqrt{13}$

c) 7 d) $4\sqrt{3}$

21. The plane which bisects the line segment joining the points (-3, -3, 4) and (3, 7, 6) at right angles, passes through which one of the following points? **[4]**

a) (2, 1, 3) b) (4,1, -2)

c) (-2, 3, 5) d) (4, - 1, 7)

22. The angle between the lines whose direction cosines are given by the equations l + m + n = 0 and 2Im + 2ln - mn = 0 is: **[4]**

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$

c) $\frac{\pi}{3}$ d) $\frac{3\pi}{4}$

23. A perpendicular is drawn from a point on the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{1}$ to the plane x + y + z = 3 such that the foot of the perpendicular Q also lies on the plane x - y + z = 3. Then, the coordinates of Q are **[4]**

a) (-1, 0, 4) b) (4, 0, -1)

c) (2, 0, 1) d) (1, 0, 2)

24. The angle between the planes 2x + y + z = 6 and a plane perpendicular to the planes 2x + 3y - z = 7 and x - y + 2z = 1 is: **[4]**

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$

c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$

25. If O is the origin and |OP| = 3 with direction ratios -1, 2, -2, then co-ordinates of P are **[4]**

a) $\left(-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right)$ b) (1, 2, 2)

c) (-3, 6, -9) d) (-1, 2, -2)

26. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to **[4]**

a) $= \frac{1}{2}(-3\hat{i} + 9\hat{j} + 5\hat{k})$ b) $3\hat{i} - 9\hat{j} - 5\hat{k}$

c) $\frac{1}{2}(3\hat{i} - 9\hat{j} + 5\hat{k})$ d) $(-3\hat{i} + 9\hat{j} + 5\hat{k})$

27. If V = $\{a\hat{i} + b\hat{j} + c\hat{k}$: a, b, c $\in$ {-1, 1}} is a set of eight vectors, then the total number of ways of selecting a set of three coplanar vectors is **[4]**

a) 32
b) 8
c) 24
d) 16

28. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = b_1\vec{i} + b_2\hat{j} + b_3\hat{k}, \vec{c} = 2\hat{i} + 2\hat{j} - i$ and $b_1 = b_2$. Then vector $\vec{b}$ satisfying $\bar{a} \times \vec{b} + \bar{c} = \vec{0}$ is **[4]**

a) $\frac{1}{4}(\hat{i} + \hat{j} - 8\hat{k})$
b) $\frac{1}{2}(\hat{i} + \hat{j} - 4\hat{k})$
c) $\frac{1}{4}(\hat{i} + \vec{j} + 8\hat{k})$
d) $\frac{1}{2}(\hat{i} + \hat{j} + 4\hat{k})$

29. If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ from the sides BC, CA and AB respectively of a $\triangle$ABC, then **[4]**

a) $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0$
b) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
c) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$
d) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a}$

30. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-collinear vectors in a plane such that $|\vec{a}|$ = 1, $|\vec{b}|$ = 3 and $|\vec{c}| = \sqrt{10}$. If $\vec{a} \times \vec{c} = \alpha$ and $\vec{b} \times \vec{c} = \beta$ where, $\alpha, \beta \in \left[\frac{\pi}{2}, \pi\right]$, then $\alpha + \beta$ equals: **[4]**

a) $\frac{3\pi}{2}$
b) $\frac{7\pi}{6}$
c) $\frac{7\pi}{4}$
d) $\frac{4\pi}{3}$

31. If $\bar{a}, \bar{b}$ are $\vec{c}$ unit vectors satisfying $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$, then $|3\vec{a} + 3\vec{b} + 5\vec{c}|$ is equal to **[4]**

a) 0
b) 3
c) 1
d) 2

32. A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k} (\alpha, \beta \in \boldsymbol{R})$ lies in the plane of the vectors, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then: **[4]**

a) $\vec{a} \cdot \hat{k} + 4 = 0$
b) $\vec{a} \cdot \hat{k} + 2 = 0$
c) $\vec{a} \cdot \hat{i} + 3 = 0$
d) $\vec{a} \cdot \hat{i} + 1 = 0$

33. If $|\vec{a}|$ = 2, $|\vec{b}|$ = 3 and $|2\vec{a} - \vec{b}|$ = 5, then $|2\vec{a} + \vec{b}|$ equals: **[4]**

a) 7
b) 1
c) 17
d) 5

34. If $\vec{a} = (\hat{i} + \hat{j} + \hat{k}), \vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then $\vec{b}$ is equal to **[4]**

a) $\hat{i} - \hat{j} + \hat{k}$
b) $\hat{i}$
c) $2\hat{i}$
d) $2\hat{j} - \hat{k}$

35. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. if $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is 30^o, then $|(\vec{\mathbf{a}} \times \vec{b}) \times \vec{c}|$ is equal **[4]**

a) $\frac{2}{3}$
b) 3
c) 2
d) $\frac{3}{2}$

36. Let $\hat{a}$ and $\hat{c}$ be unit vectors at an angle $\frac{\pi}{3}$ with each other. If $(\hat{a} \times (\hat{b} \times \hat{c})) \cdot (\hat{a} \times \hat{c})$ = 5, then the value of $[\hat{a}\hat{b}\hat{c}]$ is equal to: **[4]**

a) 5
b) 10
c) -5
d) -10

37. Let $\vec{q} = \hat{i} + \hat{j} - 2\hat{k}, \vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{p}$ is a unit vector, then the maximum value of $[\vec{p} \quad \vec{q} \quad \vec{r}]$ is **[4]**

a) 1
b) $3\sqrt{3}$
c) $3\sqrt{5}$
d) 7

38. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{(\vec{b}+\vec{c})}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is **[4]**

a) $\frac{\pi}{4}$
b) $\frac{3\pi}{4}$
c) $\frac{\pi}{2}$
d) π

39. For any three vectors $\vec{a}, \vec{b}, \vec{c}$, the value of $(\vec{a} - \vec{b}) \cdot (\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})$ is **[4]**

a) $2[\vec{a} \quad \vec{b} \quad \vec{c}]$
b) $-|\vec{a}||\vec{b}||\vec{c}|$
c) $|\vec{a}||\vec{b}||\vec{c}|$
d) 0

40. Let $\vec{a} = \sqrt{3}\hat{i} + \hat{j}, \vec{b} = \hat{i} + \sqrt{3}\hat{j}$ and $\vec{c} = 2\sqrt{2}\hat{i} - \sqrt{2}\hat{j}$ be the position vectors of A, B and C with respect to the origin O, respectively, then the distance of C from the bisector of the acute angle of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is **[4]**

a) 6 units
b) 2 units
c) 3 units
d) 1 units

41. The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is: **[4]**

a) $\sqrt{\frac{3}{2}}$
b) $\frac{\sqrt{3}}{2}$
c) $3\sqrt{6}$
d) $\sqrt{6}$

42. If $\vec{a} = 2\vec{i} + \hat{j} + 3\hat{k}, \vec{a} \times \vec{b} = \hat{i} + \hat{k}$ and $\vec{a} \cdot \vec{b} = 1$, then $|\vec{b}|$ is equal to **[4]**

a) $\sqrt{17}$
b) $\sqrt{14}$
c) $\frac{\sqrt{15}}{14}$
d) $\frac{\sqrt{17}}{14}$

43. If ABCDEF is a regular hexagon and $\overrightarrow{AB} = \vec{a}, \overrightarrow{BC} = \vec{b}$, then $\overrightarrow{CD}$ is equal to **[4]**

a) $\vec{b} - \vec{a}$
b) $\vec{a} - \vec{b}$
c) none of these
d) $\vec{a} + \vec{b}$

44. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$, then $|\vec{c}|^2$ is equal to **[4]**

a) $\frac{19}{2}$
b) 8
c) 9
d) $\frac{17}{2}$

45. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non zero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$ and if the angles between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, then $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}]^2$ is **[4]**

a) $\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$
b) $\frac{3}{4}|\vec{a}|^2|\vec{b}|^2$
c) 1
d) $3|\vec{a}|^2|\vec{b}|^2$

46. If lines x = ay + b, z = cy + d and x = a'z + b, y = c'z + d' are perpendicular, then **[4]**

a) bb' + cc' + 1 = 0
b) ab' + bc' + 1 = 0
c) aa' + c + c' = 0
d) cc' + a + a' = 0

47. Angle between vectors $\vec{a}$ and $\vec{b}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c}=\vec{0}$ **[4]**

a) $\frac{\pi}{4}$

b) $\frac{\pi}{2}$

c) $\frac{\pi}{3}$

d) $\frac{\pi}{6}$

48. Let $\vec{a}=\hat{i}+\hat{j}+\sqrt{2}\hat{k}$, $\vec{b}=b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$ and $\vec{c}=5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a}$ + $\vec{b}$ is perpendicular to $\vec{c}$, then | $\vec{b}$| is equal to **[4]**

a) 4

b) $\sqrt{22}$

c) 6

d) $\sqrt{32}$

49. If the length of the perpendicular from the point $(\beta, 0, \beta)(\beta \neq 0)$ to the line, $\frac{x}{1}=\frac{y-1}{0}=\frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then β is equal to **[4]**

a) -2

b) 2

c) 1

d) -1

50. In a parallelogram ABCD, $|\overrightarrow{\mathrm{AB}}|=\vec{a}, |\overrightarrow{\mathrm{AD}}|=\vec{b}$ and $|\overrightarrow{\mathrm{AC}}|=\vec{c}$, then $\overrightarrow{\mathrm{DB}} \cdot \overrightarrow{\mathrm{AB}}$ has the value **[4]**

a) $\frac{1}{3}\left(|\vec{b}|^2+|\vec{c}|^2-|\vec{a}|^2\right)$

b) $\frac{1}{2}\left(3|\vec{a}|^2+|\vec{b}|^2-|\vec{c}|^2\right)$

c) $\frac{1}{2}\left(|\vec{a}|^2-|\vec{b}|^2+|\vec{c}|^2\right)$

d) $\frac{1}{2}\left(|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2\right)$

51. Let $\alpha=(\lambda-2)\vec{a}+\vec{b}$ and $\beta=(4\lambda-2)\vec{a}+3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors α and β are collinear is **[4]**

a) -4

b) -3

c) 4

d) 3

52. The value of **a**, so that the volume of parallelopiped formed by $a\hat{i}+(a+1)\hat{j}+\hat{k}$, $\hat{j}+a^2\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum, is **[4]**

a) $\frac{\sqrt{3}}{2}$

b) 0

c) $\frac{3}{4}$

d) $-\frac{3}{4}$

53. Let $\vec{\mathrm{a}}=\hat{\mathrm{i}}-2\hat{\mathrm{j}}+3\hat{\mathrm{k}}; \vec{\mathrm{b}}=\hat{\mathrm{i}}+\hat{\mathrm{j}}-4\hat{\mathrm{k}}; \vec{\mathrm{c}}=4\hat{\mathrm{i}}-3\hat{\mathrm{j}}+6\hat{\mathrm{k}}; \vec{\mathrm{d}}=3\hat{\mathrm{i}}-6\hat{\mathrm{j}}-5\hat{\mathrm{k}}$ then the value of $(\vec{\mathrm{a}}\times\vec{\mathrm{b}})\cdot(\vec{\mathrm{c}}\times\vec{\mathrm{d}})$ is equal to: **[4]**

a) 392

b) 214

c) 342

d) 476

Section B

54. Find the distance of the point (2, 3, 4) from the plane 3x + 2y + 2z + 5 = 0, measured parallel to the line $\frac{x+3}{3}=\frac{y-2}{6}=\frac{z}{2}$. **[4]**

55. Find the value of m for which the line $\vec{r}=(\hat{i}+2\hat{k})+\lambda(2\hat{i}-m\hat{j}-3\hat{k})$ is parallel to the plane $\vec{r}\cdot(m\hat{i}+3\hat{j}+\hat{k})=4$. **[4]**

56. If 4x + 4y - λz = 0 is the equation of the plane through the origin that contains the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{4}$. Find the value of λ. **[4]**

57. If the origin is the centriod of a triangle ABC having vertices A (a, 1, 3), B (-2, b, -5) and C (4, 7, c), find the value of b. **[4]**

58. Write the distances of the point (7, -2, 3) from YZ-planes. **[4]**

59. Let $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a}$ = 0, then the value of $\vec{r} \cdot \vec{b}$ is: **[4]**

60. Consider the set of eight vectors V = $\{a\hat{i} + \hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is: **[4]**