RAM ♈ BARN 01

Class 10 - Mathematics

Section A

1. Draw the graphs of 2x + y = 6 and 2x - y + 2 = 0. Shade the region bounded by these lines and x-axis. Find the area of the shaded region.
2. Solve graphically system of linear equations.

   2x + y - 11=0,

   x - y -1 = 0

   Also find the coordinates of the points where the lines meet y-axis.
3. A train covered a certain distance at a uniform speed. If it were 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/h, it would have taken 6 hours more than the scheduled time. Find the length of the journey.
4. A and B are friends and their ages differ by 2 years. A's father D is twice as old as A and B is twice as old as his sister C. The age of D and C differ by 40 years. Find the ages of A and B.
5. Points A and B are 70 km. apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours, but if they travel towards each other, they meet in one hour. Find the speed of the two cars.
6. Two digit number is obtained by either multiplying the sum of digits by 8 and then subtracting 5 by multiplying the difference of digits by 16 and then adding 3. Find the number.

Section B

7. Determine whether the given quadratic equation have real roots and if so, find the roots

   $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$
8. Swati can row her boat at a speed of 5 km/hr in still water. If it takes her 1 hour more to row the boat 5.25 km upstream than to return downstream, find the speed of the stream.
9. If the roots of the quadratic equation $\left(c^2 - ab\right)x^2 - 2\left(a^2 - bc\right)x + b^2 - ac = 0$ in x are equal then show that either a = 0 or $a^3 + b^3 + c^3 = 3abc$
10. Solve for x

    $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$ where $a + b + x \neq 0$ and $a, b\ x \neq 0$
11. Find the value of m for which the quadratic equation $(m+1)y^2 - 6(m+1)y + 3(m+9) = 0$, $m \neq -1$ has equal roots. Hence find the roots of the equation.

Section C

12. AD and PM are medians of triangles ABC and PQR respectively where △ABC ∼ △PQR. Prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.
13. ABCD is a trapezium in which AB || DC and P and Q are points on AD and BC, respectively such that PQ || DC. If PD = 18 cm, BQ = 35 cm and QC = 15 cm, find AD.
14. In the given figure, DEFG is a square and $\angle BAC = 90^\circ$.

    Prove that

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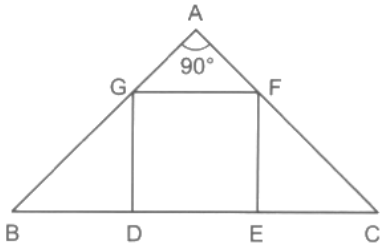
i. $\triangle AGF \sim \triangle DBG$

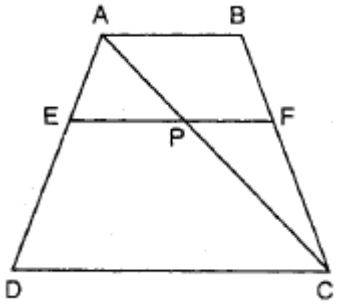
ii. $\triangle AGF \sim \triangle EFC$

iii. $\triangle DBG \sim \triangle EFC$

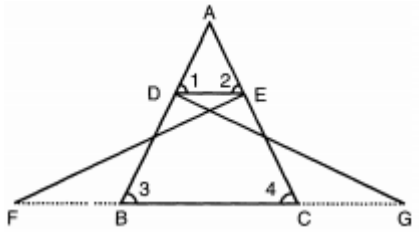
iv. $DE^2 = BD \times EC$



15. In Fig. if $EF \| DC \| AB$. prove that $\frac{AE}{ED} = \frac{BF}{FC}$.



16. In the following figure,$\triangle FEC \cong \triangle GBD$ and $\angle 1 = \angle 2$ Prove that $\triangle ADE \cong \triangle ABC$.



17. In the figure, OB is the perpendicular bisector of the line segment DE, FA $\perp$ OB and F E intersect OB at point C. Prove that $\frac{1}{\text{OA}} + \frac{1}{\text{OB}} = \frac{2}{\text{OC}}$.

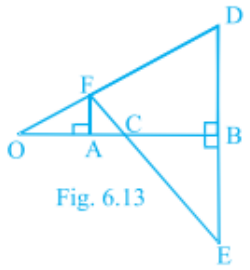
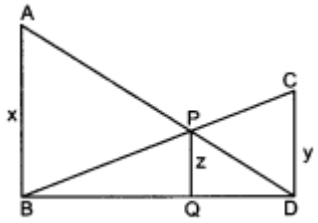


Fig. 6.13

18. In figure AB || PQ || CD, AB = x units, CD = y units and PQ = z units, prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$



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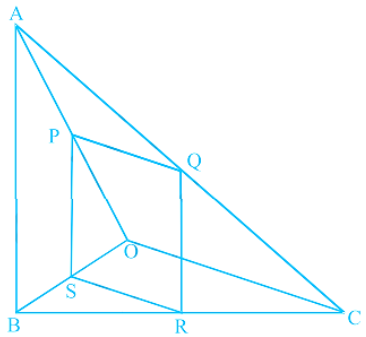
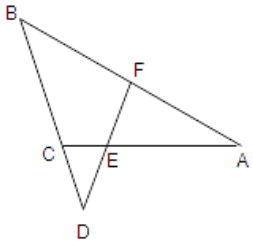
19. In the figure, if PQRS is a parallelogram and AB || PS, then prove that OC || SR.



20. In trapezium ABCD, AB || DC and DC = 2AB. EF || AB, where E and F lie on BC and AD respectively, such that $\frac{BE}{EC} = \frac{4}{3}$. Diagonal DB intersects EF at G. Prove that 7EF = 11 AB.

21. In the given figure , a $\angle AEF = \angle AFE$ and E is the mid-point of CA. Prove that $\frac{BD}{CD} = \frac{BF}{CE}$

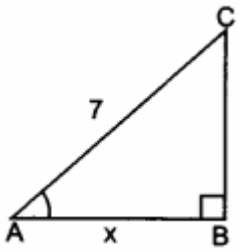


Section D

22. Prove that: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$.

23. Prove that : (sin A + sec A)^2 + (cos A + cosec A)^2 = (1 + sec A cosec A)^2.

24. In $\triangle$ABC, AB = x units, AC = 7 units, and $\angle B = 90^\circ$, cos B = 0. Evaluate : $\sqrt{7-x}$ tan C + $\sqrt{7+x}$ cot A - 14 cos A + 21 sin C + $\sqrt{49+x^2}$ cos B.



25. If a cos θ +b sin θ =m and a sin θ - b cos θ =n, prove that m^2 + n^2 = a^2 + b^2.

26. Prove that $\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{(\sec^3\theta-\csc^3\theta)} = \sin^2\theta\cos^2\theta$.

27. Prove that: $\left(\frac{1}{\sec^2\theta-\cos^2\theta} + \frac{1}{cosec^2\theta-\sin^2\theta}\right)\sin^2\theta\cos^2\theta = \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

28. Prove the trigonometric identity:

$2\sec^2\theta - \sec^4\theta - 2\cos ec^2\theta + \cos ec^4\theta = \cot^4\theta - \tan^4\theta$

29. Prove that: $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta cosec\theta - 2\sin\theta\cos\theta$

30. If x = γ cos α sin β ; y = γ cos α cos β and $z = \gamma \sin\alpha$, show that x^2 + y^2 + z^2 = γ^2.

31. Prove that : (sin θ + 1 + cos θ) (sin θ – 1 + cos θ) . sec θ cosec θ = 2

32. If a sin θ + b cos θ = c, then prove that a cos θ – b sinθ = $\sqrt{a^2+b^2-c^2}$.

33. If 2 cos θ - sin θ = x and cos θ - 3 sin θ = y, prove that 2x^2 + y^2 - 2xy = 5.

34. Prove the trigonometric identity:

If cosecθ - sinθ = a^3, secθ - cosθ = b^3, prove that a^2 b^2(a^2 + b^2) = 1

35. Find the value of θ, if $\frac{cos\theta}{1-sin\theta} + \frac{cos\theta}{1+sin\theta}$ = 4, $\theta \leqslant 90^o$.

36. Prove the following identity: $\frac{\sin A}{\sec A+\tan A-1} + \frac{\cos A}{cosec A+\cot A-1}$ = 1

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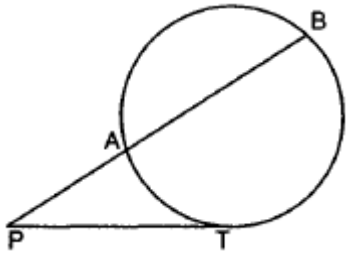
Section E

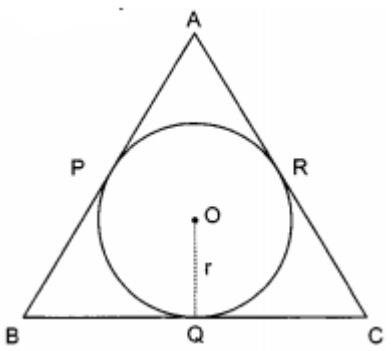
37. The angle of elevation of a cloud from a point h metres above the surface of a lake is θ and the angle of depression of its reflection in the lake is ϕ. Prove that the height of the cloud above the lake is $h\left(\frac{\tan\phi+\tan\theta}{\tan\phi-\tan\theta}\right)$
38. From the top of a hill, the angles of depression of two consecutive kilometre stones due east are found to be 45° and 30° respectively. Find the height of the hill.
39. A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30°. Another boy is standing on the roof of a 10 m high building and is flying his kite at an elevation of 45°. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.
40. An aeroplane when flying at a height of 3000 metres from the ground passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant. [Take $\sqrt{3}$=1.73]

Section F

41. In the given figure, PT is tangent to the circle at T. If PA = 4 cm and AB = 5 cm, find PT.



42. In figure the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P,Q and R respectively.



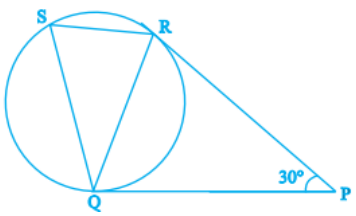
Prove that:

i. AB + CQ = AC + BQ

ii. $Area(\Delta ABC) = \frac{1}{2}(\text{Perimeter of } \Delta ABC) \times r$

43. Two tangents PA and PB are drawn to the circle with centre O, such that ∠APB = 120°. Prove that OP = 2AP.
44. If from an external point B of a circle with centre 'O', two tangents BC, BD are drawn such that ∠DBC = 120°, prove that BC + BD = BO, i.e., BO = 2BC.
45. Tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30^o$. A chord RS is drawn parallel to tangent PQ. Find $\angle RQS$.

Hint : Draw a line through Q and perpendicular to QP.



46. A is a point at a distance 13 cm from the centre 'O' of a circle of radius 5 cm. AP and AQ are the tangents to circle at P and Q. If a tangent BC is drawn at point R lying on minor arc PQ to intersect AP at B AQ at C. Find the perimeter of

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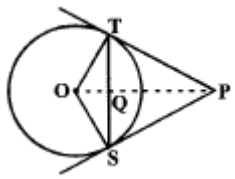
ΔABC.

47. If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that at the radius r of the circle which touches the sides of the triangle is given by r = $\frac{a+b-c}{2}$
48. In the adjoining figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If OP = 2r, show that $\angle OTS = \angle OST = 30^\circ$



Section G

49. An arc of a circle is of length 5π cm and the sector it bounds has an area of 20 πcm^2. Find the radius of the circle.
50. The radius of a circle with centre O is 7 cm. Two radii OA and OB are drawn at right angles to each other. Find the areas of minor and major segments.
51. A chord of a circle of radius 30 cm makes an angle of 60° at the centre of the circle. Find the areas of the minor and major segments. [Take π =3.14 and $\sqrt{3}$ = 1.732.]
52. The length of minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6:05 am and 6:40 am.
53. Area of a sector of central angle 200° of a circle is 770 cm^2. Find the length of the corresponding arc of this sector.

Section H

54. From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same height and same base is removed. Find the volume of the remaining solid. Also, find the whole surface area.
55. The internal and external diameters of a hollow hemispherical vessel are 21 cm and 25.2 cm respectively. The cost of painting 1 cm^2 of the surface is 10 paise. Find the total cost to paint the vessel all over.
56. Sushant has a vessel, of the form of an inverted cone, open at the top, of height 11 cm and radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put in the vessel due to which $\left(\frac{2}{5}\right)^{th}$ of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the flower beds. What value has been shown by Sushant?
57. A cone of maximum size is carved out from a cube edge 14 cm. Find the surface area of remaining solid after the cone is carved out.
58. A toy is in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere. If the radius of base of the cone is 21 cm and its volume is $\frac{2}{3}$ of the volume of the hemisphere, calculate the height of the cone and the surface area of the toy.
59. A cube of side 4 cm contains a sphere touching its sides. Find the volume of the gap in between.
60. A solid is consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm. It is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.
61. A right circular cone and a right circular cylinder have equal base and equal height. If the radius of the base and height are in the ratio 5 : 12, write the ratio of the total surface area of the cylinder to that of the cone.

Section I

62. In the following data, the median of the runs scored by 60 top batsmen of the world in one-day international cricket matches is 5000. Find the missing frequencies x and y.

Runs Scored	2500 - 3500	3500 - 4500	4500 - 5500	5500 - 6500	6500 - 7500	7500 - 8500

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Number of batsmen	5	x	y	12	6	2

63. In the following data, find the values of p and q. Also, find the median class and modal class.

Class	**Frequency(f)**	**Cumulative frequency(cf)**
100 - 200	11	11
200 - 300	12	p
300 - 400	10	33
400 - 500	q	46
500 - 600	20	66
600 - 700	14	80

64. Find the mean marks of the students for the following distribution

Marks	**Number of Students**	**Marks**	**Number of Students**
0 and above	80	60 and above	28
10 and above	77	70 and above	16
20 and above	72	80 and above	10
30 and above	65	90 and above	8
40 and above	55	100 and above	0
50 and above	43		

65. An incomplete distribution is given below:

Variable	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	12	30	-	65	-	25	18

You are given that the median value is 46 and the total number of items is 230.

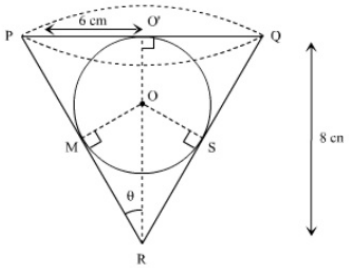
i. Using the median formula fill up missing frequencies.

ii. Calculate the AM of the completed distribution.

Section J

66. A well, whose diameter is 7m, has been dug 22.5 m deep and the earth dugout is used to form an embankment around it. If the height of the embankment is 1.5 m, find the width of the embankment.

67. A conical vessel of radius 6 cm and height 8 cm is completely filled with water. A sphere is lowered into the water and its size is such that when it touches the sides, it is just immersed as shown in Figure. What fraction of water over flows?



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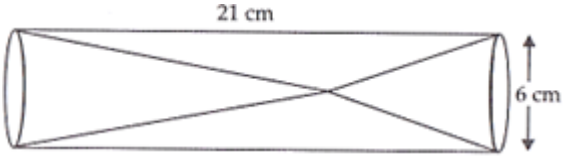
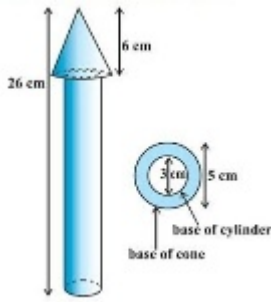
68. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite filled into it. The diameter of the pencil is 7 mm, the diameter of the graphite is 1 mm and the length of the pencil is 10 cm. Calculate the weight of the whole pencil, if the specific gravity of the wood is 0.7 gm/cm^3 and that of the graphite is 2.1 gm/ cm ^{3}.
69. Two solid cones A and B placed in a cylindrical tube as shown in the figure. The ratio of their capacities are 2 : 1. Find the heights and capacities of cones. Also, find the volume of the remaining portion of the cylinder.



70. A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical ball is dropped into the tub and the level of the water is raised by 6.75 cm. Find the radius of the ball.
71. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. Also, find the total surface area of the toy.
72. A rocket is in the form of a circular cylinder closed at the lower end with a cone of the same radius attached to the top. The cylinder is of radius 2.5 m and height 21 m and the cone has the slant height 8 m. Calculate the total surface area of the rocket.
73. From a cubical piece of wood of side 21 cm, a hemisphere is carved out in such a way that the diameter of the hemisphere is equal to the side of the cubical piece. Find the surface area and volume of the remaining piece.
74. A wooden toy rocket is in the shape of a cone mounted on a cylinder as shown in given below figure. The height of the entire rocket is 26 cm, while the height of the conical part is 6 cm. The base of the conical portion has a diameter of 5 cm, while the base diameter of the cylindrical portion is 3 cm. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take π =3.14)



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