Hitesh sir classes (maths)

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PAPER 04 (2023)

Class 12 - Mathematics

Time Allowed: 2 hours **Maximum Marks: 60**

General Instructions:

All questions are compulsory

All the very best

Read -Think -Believe and then solve

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Section A

1. If the set A contains 5 elements and the set B contains 6 elements, then the number of one – one and onto mappings from A to B is **[1]**

   a) none of these b) 720

   c) 120 d) 0

2. The value of $\sin^{-1}\left(\cos\left(\frac{43\pi}{5}\right)\right)$ is **[1]**

   a) $-\frac{\pi}{10}$ b) $\frac{-7\pi}{5}$

   c) $\frac{3\pi}{5}$ d) $\frac{\pi}{10}$

3. If A is 3×4 matrix and B is a matrix such that A^T B and BA^T are both defined. Then, B is of the type **[1]**

   a) 4×4 b) 4×3

   c) 3×3 d) 3×4

4. Let A be a square matrix of order 3. If det. A = 2 then the value of det. (adj. A^3) is equal to: **[1]**

   a) 2^6 b) 2^9

   c) 2^3 d) 2^{12}

5. If $f(x) = \begin{cases} \frac{\sin(\cos x)-\cos x}{(\pi-2x)^2} & , x \neq \frac{\pi}{2} \\ k & , x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then k is equal to **[1]**

   a) 1 b) -1

   c) 0 d) $\frac{1}{2}$

6. Let f(x) = $(\sin(\tan^{-1} x) + \sin(\cot^{-1} x))^2 - 1$, |x| > 1. If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}$ ($\sin^{-1}$ (f(x)) and y $(\sqrt{3}) = \frac{\pi}{6}$, they y $(-\sqrt{3})$ is equal to: **[1]**

   a) $\frac{5\pi}{6}$ b) $\frac{\pi}{3}$

   c) -$\frac{\pi}{6}$ d) $\frac{2\pi}{3}$

7. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4), and (0, 5). If the maximum value of z = ax + by, where a, b > 0 occurs at both (2, 4) and (4, 0), then: **[1]**

a) 3a = b

b) 2a = b

c) a = 2b

d) a = b

Section B

8. Let A = R - {2} and B = R - {1}. If f : A $\rightarrow$ B is a mapping defined by $f(x) = \frac{x-1}{x-2}$, Show that f is bijective. **[2]**

9. Find the minimum value of n for which $\tan^{-1}\frac{n}{\pi} > \frac{\pi}{4}, n \in N$. **[2]**

10. If A =[a_{ij}] = $\begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and B = $[b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$ then find a_{11} b_{11} + a_{22} b_{22} **[2]**

11. Evaluate the determinant $\begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}$ by expanding it along first column. **[2]**

12. If the system of equations **[2]**

x = cy + bz

y = az + cx

z = bx + ay

has a non-trivial solution, show that $a^2 + b^2 + c^2 + 2abc = 1$

13. If y = $a^x + e^x + x^x + x^a$, find $\frac{dy}{dx}$ at x = a **[2]**

14. Let I be any interval disjoint from [-1, 1]. Prove that the function f given by $f(x) = x + \frac{1}{x}$ is increasing on I. **[2]**

15. Separate [0, π/2] into subintervals in which f(x) = sin 3x is increasing or decreasing. **[2]**

16. Prove that $\int\limits_0^\pi \sin^{2m} x \cos^{2m+1} x dx = 0$, where m is a positive integer. **[2]**

17. Integrate the function $e^{3\log x}\left(x^4+1\right)^{-1}$ **[2]**

18. Find the general solution of the differential equation: sin $\frac{dy}{dx}$ + (cos x)y = cos x $\sin^2$x **[2]**

Section C

19. Find the local maxima and local minima, if any. Find also the local maximum and the local minimum values, as the case may be: f(x) = $\sin^4$ x + $\cos^4$ x, $0 < x < \frac{\pi}{2}$ **[3]**

20. If [•] denotes the greatest integer function, then find the value of $\int_1^2 [3x]dx$ **[3]**

Section D

21. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. **[5]**

22. Evaluate the definite integral $\int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx$. **[5]**

23. Evaluate $\int_0^\pi \frac{x}{a^2\cos^2 x + b^2 \sin^2 x} dx$. **[5]**

24. Find a particular solution of the differential equation $\frac{dy}{dx} + y\cot x = 4x.\cos ec\, x$ **[5]**

Given that y = 0 when $x = \frac{\pi}{2}$

25. Show that the minimum of Z occurs at more than two points. **[5]**

Maximize Z = x + y subject to $x - y \leq -1, -x + y \leq 0, x, y \geq 0$.