



PAPER 03 (2023)

Class 12 - Mathematics

Time Allowed: 1 hour and 30 minutes

Maximum Marks: 60

General Instructions:

All questions are compulsory

wish you All the very best

Read -Think -Believe and then solve

Section A

1. If  $y = \frac{\log x}{x}$ , then  $\frac{d^2y}{dx^2} =$  [1]
  - a)  $\frac{2\log x - 3}{x^3}$
  - b) None of these
  - c)  $\frac{2\log x - 3}{x^4}$
  - d)  $\frac{3 - 2\log x}{x^3}$
2. If  $\sin(x + y) = \log(x + y)$ , then  $\frac{dy}{dx} =$  [1]
  - a) -2
  - b) -1
  - c) 2
  - d) 1
3. The least value of k for which the function f defined as  $f(x) = x^2 + kx + 1$ , is an increasing function in the interval (1, 2), is: [1]
  - a) -1
  - b) -2
  - c) -3
  - d) -4
4. A point on the curve  $6y = x^3 + 2$  at which its ordinate is changing 8 times as fast as its abscissa is: [1]
  - a) (4, 11)
  - b) (4, -11)
  - c) (-4, 11)
  - d) (-4, -11)
5. The value of  $\int_0^\pi \frac{1}{5+3\cos x} dx$  is [1]
  - a) 0
  - b)  $\frac{\pi}{8}$
  - c)  $\frac{\pi}{4}$
  - d)  $\frac{\pi}{2}$
6. What is integrating factor of  $\frac{dy}{dx} + y \sec x = \tan x$ ? [1]
  - a)  $\sec x$
  - b)  $e^{\sec x}$
  - c)  $\sec x + \tan x$
  - d)  $\log(\sec x + \tan x)$
7. What is the order of the differential equation  $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - \sin^2 y = 0$ ? [1]
  - a) undefined
  - b) 3
  - c) 2
  - d) 1

### Section B

8. If  $xy^y + y^x = (x + y)^{x+y}$ , find  $\frac{dy}{dx}$  [3]
9. A given quantity of metal is to be cast into a half-cylinder with a rectangular base and semicircular ends. Show that in order that the total surface area may be minimum, the ratio of the length of the cylinder to the diameter of its semicircular ends is  $\pi: (\pi + 2)$ . [3]
10. Find  $\int \frac{x^3}{x^4 + 3x^2 + 2} dx$ . [3]
11. Evaluate:  $\int_{-1}^1 |x \cos \pi x| dx$  [3]
12. Solve the differential equation:  $\sin^4 x \frac{dy}{dx} = \cos x$  [3]
13. Find the general solution of the differential equation:  $(x + y + 1) \frac{dy}{dx} = 1$  [3]
14. If  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ , find  $\frac{dy}{dx}$  and  $\frac{dx}{dy}$ . Also, show that  $\frac{dy}{dx} \cdot \frac{dx}{dy} = 1$ . [5]
15. Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a volume as large as possible, when revolved about one of its sides. Also, find the maximum volume. [5]
16. Of all the closed right circular cylindrical cans of volume  $128\pi \text{ cm}^3$ , find the dimensions of the can which has minimum surface area. [5]
17. Evaluate  $\int (\sqrt{\cot x} + \sqrt{\tan x}) dx$ . [5]
18. Evaluate:  $\int \frac{1}{\sin x + \sqrt{3} \cos x} dx$  [5]
19. Evaluate:  $\int \frac{1}{13 + 3 \cos x + 4 \sin x} dx$  [5]
20. Find the particular solution of the differential equation  $(\tan^{-1} y - x) dy = (1 + y^2) dx$ , given that when  $x = 0$ ,  $y = 0$ . [5]