Hitesh sir classes (maths)

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PAPER 02 (2023)

Class 12 - Mathematics

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 50**

General Instructions:

All questions are compulsory

Wish you all the best

Read -Think -Believe and then solve

Section A

1. Let A = $\{x_1, x_2, \ldots\ldots\ldots, x_7\}$ and B = $\{y_1, y_2, y_3\}$ be two sets containing seven and three distinct elements respectively. Then the total number of functions f : A $\rightarrow$ B that are onto, if there exist exactly three elements x in A such that $f(x) = y_2$, is equal to: **[1]**

   a) $14\ {}^7C_2$
   b) $12\ {}^7C_2$
   c) $16\ {}^7C_3$
   d) $14\ {}^7C_3$

2. Let P be the relation defined on the set of all real numbers such that P = $\{(a, b): \sec^2 a - \tan^2 b = 1\}$. Then P is: **[1]**

   a) symmetric and transitive but not reflexive
   b) an equivalence relation
   c) reflexive and symmetric but not transitive
   d) reflexive and transitive but not symmetric

3. Let f : D $\rightarrow$ R be defined as $f(x) = \frac{x^2+2x+a}{x^2+4x+3a}$ where D and R denote the domain of f and the set of all real numbers respectively. If f is surjective mapping then the range of a is: **[1]**

   a) $0 \leq a \leq 1$
   b) $0 \leq a < 1$
   c) $0 < a \leq 1$
   d) $0 < a < 1$

4. The greatest and least values of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ are respectively **[1]**

   a) $\frac{5\pi^2}{4}$ and $\frac{\pi^2}{8}$
   b) $\frac{\pi}{2}$ and $\frac{-\pi}{2}$
   c) $\frac{\pi^2}{4}$ and 0
   d) $\frac{\pi^2}{4}$ and $\frac{-\pi^2}{4}$

5. The value of $\sin(2\sin^{-1}(0.6))$ is **[1]**

   a) 0.96
   b) 0.48
   c) sin 1.2
   d) 1.2

6. If A, B are square matrices of order 3, A is non-singular and AB = 0, then B is a **[1]**

   a) non-singular matrix
   b) null matrix
   c) singular matrix
   d) unit matrix

7. If P = $\begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and Q = PAP^T, then $P^TQ^{2005}P$ is **[1]**

a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2005 \\ 2005 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 \\ 2005 & 1 \end{bmatrix}$

8. The system of equations, 3x + y – z = 0, 5x + 2y – 3z = 2, 15x + 6y – 9z = 5 has **[1]**

a) a unique solution

b) two distinct solutions

c) no solution

d) infinitely many solutions

9. Let M = $\begin{bmatrix} x & 2x & 3x \\ f(x) & g(x) & h(x) \\ 0 & 1 & 1 \end{bmatrix}$ be a singular matrix. If $f(x) = \ln(e^x + 1)$ and g(x) = ln(e^x - 1), then the value of h'(In3) is: **[1]**

a) 3

b) $\frac{9}{4}$

c) $\frac{9}{8}$

d) 6

10. A speaks truth in 84% cases and B speaks truth in 96% cases. The probability that both do not contradict each other in a statement is: **[1]**

a) $\frac{504}{625}$

b) $\frac{4}{625}$

c) $\frac{117}{625}$

d) $\frac{508}{625}$

11. If A and B are two independent events such that P(A) > 0, and P(B) $\neq$ 1. then $P(\bar{A}/\bar{B})$ is equal to **[1]**

a) $1 - P(A/\bar{B})$

b) $\frac{P(\bar{A})}{P(\bar{B})}$

c) $\frac{1-P(A \cup B)}{P(B)}$

d) 1 - P(A/B)

12. Let A and B be two events such that the probability that exactly one of them occurs $\frac{2}{5}$ is and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is: **[1]**

a) 0.10

b) 0.20

c) 0.02

d) 0.01

Section B

13. Classify the function f : R $\to$ R, defined by $f(x) = \frac{x}{x^2+1}$ as injection, surjection or bijection. **[3]**

14. Let A = {1, 2, 3, ...9} and R be the relation in A$\times$A defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in A$\times$A. Prove that R is an equivalence relation and also obtain the equivalence class [(2, 5)]. **[5]**

15. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is divisible by 2} is an equivalence relation. Write all the equivalence classes of R. **[5]**

16. Let A = $\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$. Find matrices X and Y such that X + Y = A, where X is a symmetric and Y is a skew-symmetric matrix. **[5]**

17. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations 2x - 3y + 5z = 11; 3x + 2y - 4z = -5; x + y - 2z = -3 **[5]**

18. If a,b and c are real numbers and $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+a & c+a \end{vmatrix} = 0$ Show that either a + b + c = 0 or a = b = c **[5]**

19. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i. e if a healthy person is test then with probability 0.005 the test will imply he has the disease) If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive. **[5]**

20. If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$, then find P(A) and P(B). **[5]**