Hitesh sir classes (maths)

9717101190

PAPER 01 (2023)

Class 12 - Mathematics

Time Allowed: 1 hour and 45 minutes **Maximum Marks: 60**

General Instructions:

All questions are compulsory.

Wish you All the best.

Read -Think -Believe and then solve.

Section A

1. If y = $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

   a) $\frac{a}{b}$ b) 1

   c) -1 d) $\frac{-b}{a}$

2. If $f(x) = \frac{1}{1-x}$, then the set of points of discontinuity of the function $f(f(f(x)))$ is **[1]**

   a) {-1, 1} b) none of these

   c) {1} d) {0, 1}

3. The values of p for which the function f(x) = $\left(\frac{\sqrt{p+4}}{1-p} - 1\right)$ x^5 - 3x + ln 5 decreases for all real x is: **[1]**

   a) $[1, \infty)$ b) $(-\infty, \infty)$

   c) $\left[-4, \frac{3-\sqrt{21}}{2}\right] \cup (1, \infty)$ d) $\left[-3, \frac{5-\sqrt{27}}{2}\right] \cup (2, \infty)$

4. The function f given by f(x) = cos x + cos $\sqrt{3}$x **[1]**

   a) attains its global maximum at x = 0 b) is odd

   c) is periodic d) is not differentiable

5. If f(x) = $x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$ **[1]**

   a) f(x) is strictly increasing function b) f(x) is bounded

   c) f(x) has a local maxima d) f(x) is strictly decreasing function

6. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$, is **[1]**

   a) 2π b) $\frac{\pi}{2}$

   c) $a\pi$ d) π

7. Let $\frac{d}{dx}$[f(x)] = $\frac{e^{\sin x}}{x}$, where x > 0. If $\int_1^4 \frac{2}{x} e^{\sin x^2}$ dx = f(k) - f(1), then one of the possible value of k is **[1]**

   a) 16 b) 64

   c) 128 d) 4

8. Let f(a) = $\int_0^a \ln(1 + \tan a \tan x)\,dx$, then $f'\left(\frac{\pi}{4}\right)$ equals: **[1]**

a) $\frac{\pi}{2}+\frac{\ln 2}{2}$

b) $\frac{\pi}{4}+\ln 2$

c) $\frac{\pi}{2}+\ln 2$

d) $\frac{\pi}{4}+\frac{\ln 2}{2}$

9. The slope of the tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ is given by $\frac{y}{x}$ - $\cos^2(\frac{y}{x})$, then the equation of the curve is: **[1]**

a) $y = x \tan^{-1}[\log(\frac{x}{e})]$

b) None of these

c) $y = \tan^{-1}[\log(\frac{e}{x})]$

d) $y = x \tan^{-1}[\log(\frac{e}{x})]$

Section B

10. Prove that: $\int_0^1 \frac{\log(1+x)}{(1+x^2)}dx = \frac{\pi}{8}(\log 2)$. **[3]**

11. Integrate the function $\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$ **[3]**

12. If $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ **[5]**

13. Find the values of p and q so that $f(x) = \begin{cases} x^2+3x+p, \, if\, x \leqslant 1 \\ qx+2, \, if\, x > 1 \end{cases}$ is differentiable at x = 1. **[5]**

14. Tangent to the circle $x^2 + y^2 = a^2$ at any point on it in the first quadrant makes intercepts OA and OB on x and y axes respectively, O being the centre of the circle. Find the minimum value of OA + OB. **[5]**

15. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum, when the angle between them is $\frac{\pi}{3}$. **[5]**

16. A wire of length 36 cm is cut into two pieces. One of the pieces is turned in the form of a square and the other in the form of an equilateral triangle. Find the length of each piece so that the sum of the areas of the two be minimum. **[5]**

17. Evaluate: $\int \frac{\sqrt{x^2+1}\left(\log|x^2+1|-2\log|x|\right)}{x^4}dx.$ **[5]**

18. Evaluate: $\int \frac{1}{\sin x - \sin 2x}dx$ **[5]**

19. Solve the following differential equation. **[5]**

$\sqrt{1+x^2+y^2+x^2y^2}+xy\frac{dy}{dx}=0$

20. Solve the differential equation, $ye^{x/y}\,dx = (xe^{x/y} + y^2)\,dy$, $y \neq 0$ **[5]**