Hitesh sir classes (maths)

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ONE DAY BEFORE EXAM (2023) ALL THE VERY BEST

Class 12 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If 1, 2, 2 and 2, 1, 1 are the direction ratios of two lines and Q is the angle between the lines such that sin 2θ = $\frac{\lambda}{\sqrt{2}}$ for a scalar λ, then λ is: **[1]**

   a) $\frac{3}{4}$ b) $\frac{4}{3}$

   c) $\frac{4}{\sqrt{3}}$ d) $\frac{2}{3}$

2. The integral $\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$ is equal to (where C is a constant of integration) **[1]**

   a) $\frac{x^4}{6(2x^4+3x^2+1)^3}+C$ b) $\frac{x^{12}}{6(2x^4+3x^2+1)^3}+C$

   c) $\frac{x^4}{(2x^4+3x^2+1)^3}+C$ d) $\frac{x^{12}}{(2x^4+3x^2+1)^3}+C$

3. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}.\vec{b} \geqslant 0$ only when **[1]**

   a) $0 < \theta < \frac{\pi}{2}$ b) $0 \leqslant \theta \leqslant \pi$

   c) $0 < \theta < \pi$ d) $0 \leqslant \theta \leqslant \frac{\pi}{2}$

4. The area of the region bounded by the curves y = |x - 2|, x = 1, x = 3 and the x – axis is **[1]**

   a) 1 b) 4

   c) 2 d) 3

5. The area bounded by the curve y = x (x – 1) (x – 2) and the x-axis is equal to **[1]**

   a) $\frac{1}{2}$sq.units b) none of these

   c) 1 sq.units d) $\frac{1}{4}$sq.units

6. A speaks the truth in 70% cases and B in 80% cases. The probability that they will contradict each other in **[1]**

describing a single event is:

a) 0.56
b) 0.4
c) 0.42
d) 0.38

7. Two persons A and B appear in an interview for two vacancies. If the probabilities of their selection are $\frac{1}{4}$ and $\frac{1}{6}$ respectively, then the probability that none of them is selected is **[1]**

a) $\frac{19}{12}$
b) $\frac{5}{12}$
c) $\frac{5}{8}$
d) $\frac{1}{24}$

8. Which of the following is not a convex set? **[1]**

a) [(x, y) : 2x + 5Y < 7]
b) $\{(x, y) : x^2 + y^2 \leq 4\}$
c) {X: |X| = 5}
d) {(x, y) : $3x^2 + 2y^2 \leq 6$)

9. If $\vec{a},\ \vec{b}$ are the vectors forming consecutive sides of a hexagon ABCDEF, then the vector representing side CD is **[1]**

a) $\vec{a} - \vec{b}$
b) $\vec{b} - \vec{a}$
c) $\vec{a} + \vec{b}$
d) $-(\vec{a} + \vec{b})$

10. General solution of $ydx + (x - y^3)dy = 0$ is **[1]**

a) $xy = \frac{y^4}{4} + C$
b) None of these
c) xy = 3y + C
d) $xy = \frac{y^4}{3} + C$

11. What is the number of arbitrary constants in the particular solution of differential equation of third order? **[1]**

a) 3
b) 1
c) 0
d) 2

12. If f(a + b - x) = f (x), then $\int_a^b xf(x)dx$ is equal to **[1]**

a) $\frac{a+b}{2}\int_a^b f(x)dx$
b) $\frac{b-a}{2}\int_a^b f(x)dx$
c) $\frac{a+b}{2}\int_a^b f(b-x)dx$
d) $\frac{a+b}{2}\int_a^b f(b+x)dx$

13. The general solution of e^x cos y dx - e^x sin y dy = 0 is : **[1]**

a) e^x = c cos y
b) e^x = c sin y
c) e^x cos y = c
d) e^x sin y = c

14. If x = f (t) cos t - f '(t) sint and y = f (t) sin t + f '(t) cost, then $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$ = **[1]**

a) none of these
b) f (t) - f '' (t)
c) $\{f(t) + f''(t)\}^2$
d) $\{f(t) - f''(t))\}^2$

15. If A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-50} when $\theta = \frac{\pi}{12}$ is equal to **[1]**

a) $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$
b) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$
c) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$
d) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

16. The general solution of a differential equation of the type $\frac{dx}{dy} + \mathrm{P}_1 x = \mathrm{Q}_1$ is ? **[1]**

a) $xe^{\int \mathrm{P}_1 dy} = \int \left(\mathrm{Q}_1 e^{\int \mathrm{P}_1 dy}\right) dy + \mathrm{C}$

b) $ye^{\int \mathrm{P}_1 dy} = \int \left(\mathrm{Q}_1 e^{\int \mathrm{P}_1 dy}\right) dy + \mathrm{C}$

c) $y \cdot e^{\int_P dx} = \int \left(Q_1 e^{\int P_1 dx}\right) dx + C$

d) $xe^{\int P^1 dx} = \int \left(Q_1 e^{\int P_1 dx}\right) dx + C$

17. Let $\theta = \sin^{-1} (\sin (-600°))$, then value of θ is **[1]**

a) $\frac{-2\pi}{3}$

b) $\frac{2\pi}{3}$

c) $\frac{\pi}{2}$

d) $\frac{\pi}{3}$

18. Find the cartesian equation of the line that passes through the origin and (5, – 2, 3). **[1]**

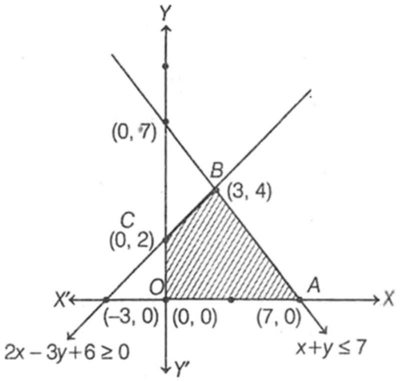
a) $\frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$

b) $\frac{x}{6} = \frac{y}{-2} = \frac{z}{3}$

c) $\frac{x}{5} = \frac{y}{-1} = \frac{z}{3}$

d) $\frac{x}{5} = \frac{y}{-2} = \frac{z}{4}$

19. **Assertion (A):** Objective function Z = 13x - 15y is minimised, subject to the constraints $x + y \leq 7$, $2x - 3y + 6 \geq 0$, $x \geq 0$, $y \geq 0$. **[1]**



The minimum value of Z is - 21.

Reason (R): Optimal value of an objective function is obtained by comparing value of objective function at all comer points.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** $\frac{d}{dx}\left(\sqrt{e^{\sqrt{x}}}\right) = \frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}$. **[1]**

Reason (R): $\frac{d}{dx}$[log (log(x))] = $\frac{1}{x \log x}$, x > 1

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Solve: $\frac{dy}{dx} = e^{x+y}$ **[2]**

22. Differentiate the function with respect to x: $\sin(x^x)$ **[2]**

23. Find the image of the point (2, -1,5) in the line $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$ **[2]**

OR

Find the angle between the pairs of lines:

$\frac{x-2}{3} = \frac{y+3}{-2}, z = 5$ and $\frac{x+1}{1} = \frac{2y-3}{3} = \frac{z-5}{2}$

24. A husband and wife appear in an interview for two vacancies for the same post. The probability of husband's **[2]**

selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. What is the probability that only one of them will be selected?

25. For the principal values, evaluate $\sin^{-1}\left[\cos\left\{2cosec^{-1}(-2)\right\}\right]$ **[2]**

Section C

26. Maximise the function Z = 11x + 7y, subject to the constraints: $x \leqslant 3, y \leqslant 2, x \geqslant 0, y \geqslant 0$. **[3]**

27. Find the area enclosed between the curve y = x^3 and the line y = x. **[3]**

OR

Sketch the region {(x, y): $9x^2 + 4y^2 = 36$} and find the area of the region enclosed by it, using integration.

28. Evaluate $\int \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ **[3]**

OR

Evaluate: $\int \frac{1}{x(x^3+8)} dx$

29. Find the foot of the perpendicular drawn from the point $\hat{i} + 6\hat{j} + 3\hat{k}$ to the line $\vec{r} = \hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$. Also, find the length of the perpendicular. **[3]**

OR

Find the angle between the lines whose direction cosines are given by the equations l + 2m + 3n = 0 and 3 Im - 4 In + mn = 0

30. Find the area of the region bounded by the parabola y^2 = 4ax and the line x = a **[3]**

31. If y= log[x + $\sqrt{x^2 + a^2}$], then show that $\left(x^2 + a^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$. **[3]**

Section D

32. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, then show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$. **[5]**

33. Given $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ find AB and use this result in solving the following system of equation. **[5]**

x - y + z = 4

x - 2y - 2z = 9

2x + y + 3z = 1

OR

An amount of Rs 5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is Rs 358. If the combined income from the first two investments is Rs 70 more than the income from the third, find the amount of each investment by matrix method.

34. Show that the function f : R → R defined by $f(x) = \frac{x}{x^2+1}$, $\forall x \in R$, is neither one-one nor onto. **[5]**

OR

Let R be relation defined on the set of natural number N as follows:

R = {(x, y): x ∈ N, y ∈ N, 2x + y = 41}. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

35. Evaluate: $\int \frac{(3\sin x - 2)\cos x}{5 - \cos^2 x - 4\sin x} dx$ **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Mrs. Maya is the owner of a high-rise residential society having 50 apartments. When he set rent at ₹10000/month, all apartments are rented. If he increases rent by ₹250/ month, one fewer apartment is rented.

The maintenance cost for each occupied unit is ₹500/month.

(i) If P is the rent price per apartment and N is the number of rented apartments, then find the profit.

(ii) If x represents the number of apartments which are not rented, then express profit as a function of x.

(iii) Find the number of apartments which are not rented so that profit is maximum.

OR

Verify that profit is maximum at critical value of x by second derivative test.

37. **Read the text carefully and answer the questions:** **[4]**

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.


(i) Write the matrix summarizing sales data of 2019 and 2020.

(ii) Find the matrix summarizing sales data of 2020.

(iii) Find the total number of cars sold in two given years, by each dealer?

OR

If each dealer receives a profit of ₹ 50000 on sale of a Hatchback, ₹100000 on sale of a Sedan and ₹200000 on sale of an SUV, then find the amount of profit received in the year 2020 by each dealer.

38. **Read the text carefully and answer the questions:** **[4]**

To hire a marketing manager, it's important to find a way to properly assess candidates who can bring radical changes and has leadership experience.

Ajay, Ramesh and Ravi attend the interview for the post of a marketing manager. Ajay, Ramesh and Ravi chances of being selected as the manager of a firm are in the ratio 4 : 1 : 2 respectively. The respective probabilities for them to introduce a radical change in marketing strategy are 0.3, 0.8, and 0.5. If the change does take place.

(i) Find the probability that it is due to the appointment of Ajay (A).

(ii) Find the probability that it is due to the appointment of Ramesh (B).