VECTOR & 3D ABHYAS 01

Class 12 - Mathematics

Section A

1. ABCD is a parallelogram with AC and BD as diagonals. Then, $\overrightarrow{AC} - \overrightarrow{BD} =$

a) $3\overrightarrow{AB}$ b) $\overrightarrow{AB}$

c) $4\overrightarrow{AB}$ d) $2\overrightarrow{AB}$

2. $[\hat{i} \quad \hat{j} \quad \hat{k}] = ?$

a) 3 b) 1

c) 2 d) 0

3. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ *with* $\hat{i}$, $\frac{\pi}{4}$ *with* $\hat{j}$ *and an acute angle* θ *with* $\hat{k}$, then the components of $\vec{a}$ are

a) $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{3}$ b) $\frac{1}{3}, \frac{1}{\sqrt{2}}, \frac{1}{2}$

c) $\frac{1}{3}, \frac{1}{\sqrt{3}}, \frac{1}{2}$ d) $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$

4. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a}.\vec{c} = 4$, then $|\vec{c}|^2$ is equal to

a) $\frac{19}{2}$ b) 8

c) 9 d) $\frac{17}{2}$

5. The vector $2\hat{j} - \hat{k}$ lies

a) in the plane of XZ b) along the X-axis

c) in the plane of XY d) in the plane of YZ

6. Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + \left(\lambda^2 - 1\right)\hat{k}$ be coplanar vectors. Then, the non-zero vector $\vec{a} \times \vec{c}$ is

a) $-10\hat{\mathrm{i}} + 5\hat{\mathrm{j}}$ b) $-14\hat{i} + 5\hat{j}$

c) $-10\hat{i} - 5\hat{j}$ d) $-14\hat{i} - 5\hat{j}$

7. The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ forming a right-handed system is

a) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$ b) $-\hat{k}$

c) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ d) $\hat{k}$

8. $[\vec{a}\ \vec{b}\ \vec{a} \times \vec{b}] + (\vec{a} \cdot \vec{b})^2 =$

a) $|\vec{a}|^2 + |\vec{b}|^2$ b) $2|\vec{a}|^2 + |\vec{b}|^2$

c) $\overrightarrow{\mathrm{a}} + \overrightarrow{\mathrm{b}}^2$ d) $\overrightarrow{|\mathrm{a}|^2}\overrightarrow{|\mathrm{b}|^2}$

9. The value of $(\vec{a} \times \vec{b})^2$ is

a) $|\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$ b) $|\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a} \cdot \vec{b})$

c) $|\vec{a}|^2 + |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$ d) $|\vec{a}|^2 + |\vec{b}|^2 - \vec{a} \cdot \vec{b}$

10. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then Let $\vec{a} + \vec{b}$ is a unit vector if

a) $\theta = \frac{\pi}{3}$ b) $\theta = \frac{2\pi}{3}$

c) $\theta = \frac{\pi}{2}$ d) $\theta = \frac{\pi}{4}$

Section B

11. If $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ, so that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.
12. Find the unit vector in the direction of the sum of the vectors $\overrightarrow{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}, \vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$
13. If $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, \left|\vec{b}\right| = 5, |\vec{c}| = 7$ find the angle between $\vec{a}$ and $\vec{b}$
14. Vectors drawn from the origin O to the points A, B and C are respectively $\vec{a}, \vec{b}$ and $4\vec{a} - 3\vec{b}$. Find $\overrightarrow{AC}$ and $\overrightarrow{BC}$.
15. Find the values of x and y if the vectors $\vec{a} = 3\hat{i} + x\hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + y\hat{k}$ are mutually perpendicular vectors of equal magnitude.
16. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{c} = \hat{j} - \hat{k}$ are given vectors, then find a vector $\vec{b}$ satisfying the equations $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a}.\vec{b} = 3$

Section C

17. If the lines x = 1 - 3λ, y = 2 + kλ, z = 3 + 2λ and x = 1 + 3kλ, y = 5 + λ, z = 6 - 5λ. are at right angles, then k is equal to:

a) $\frac{5}{4}$ b) $-\frac{10}{7}$

c) $\frac{10}{7}$ d) $-\frac{5}{4}$

18. If a vector makes angles α, β and γ with the x axis, y axis and z axis respectviely then the value of $\left(\sin^2\alpha + \sin^2\beta + \sin^2\gamma\right)$ is

a) 2 b) 0

c) 3 d) 1

19. ABCD is a parallelogram. The position vector of the points A, B and C are $3\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - 4\hat{k}$ and $-\hat{i} + \hat{j} + 2\hat{k}$ respectively. The equation of line BD is:

a) $\hat{i} - 2\hat{j} + 8\hat{k} + \lambda(3\hat{j} + 4\hat{k})$ b) $\hat{i} + 2\hat{j} - 4\hat{k} - 4\lambda(\hat{j} - 3\hat{k})$

c) $\hat{i} - 2\hat{j} + 8\hat{k} - 4\lambda(3\hat{i} - 4\hat{k})$ d) $\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(4\hat{j} + 12\hat{k})$

20. The straight line $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{0}$ is

a) parallel to the y-axis b) perpendicular to the z-axis

c) parallel to the x-axis d) parallel to the z-axis

21. The straight line $\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$

a) is parallel to z-axis b) lies in yz plane

c) lies in xy plane d) lies in zx plane

22. If the lines $\frac{x-1}{2} = \frac{y-k}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{k}$ = z intersect, then the value of k is:

a) 1 b) 4

c) -2 d) 2

Section D

23. Find the shortest distance between the following lines.
    $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{-1}, \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$
24. Find the shortest distance between the pairs of lines whose vector equations are:
    $\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$ and $\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$
25. Find the co-ordinates of the foot of perpendicular drawn from the point A (1, 8, 4) to the line joining the points B (0, –1, 3) and C (2, –3, –1).
26. Find the distance between the line $\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j})$ and the line passing through (0, -1, 2) and (1, -2, 3).
27. Find the vector and cartesian equations of the line passing through the point (2,1,3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.
28. Find the vector equation of the line passing through the point A(2, -1, 1), and parallel to the line joining the points B(-1, 4,1) and C(l, 2, 2). Also, find the Cartesian equations of the line.
29. The cartesian equation of a line is 6x -2 = 3y+1 = 2z-2 . Find the direction cosines of the line. Write down the cartesian and vector equations of a line passing through (2, -1, -1) which is parallel to the given line?
30. Find the image of the point (1, 6, 3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.