ONE DAY BEFORE (EXAM 2023)

Class 12 - Mathematics

Section A

1. If N denotes the set of all natural numbers and R be the relation on N $\times$ N defined by (a, b) R (c, d), if ad(b + c) = bc(a + d). Show that R is an equivalence relation.
2. Let the function f : R $\rightarrow$ R be defined by f(x) = cosx, $\forall$x $\in$ R. Show that f is neither one-one nor onto.
3. Let S be the set of all sets and let R = {(A, B): A $\subset$ B)}, i.e., A is a proper subset of B.
   show that R is
   i. transitive
   ii. not reflexive
   iii. not symmetric.
4. Show that the function f in A = R - $\left\{\frac{2}{3}\right\}$ defined as f (x) = $\frac{4x+3}{6x-4}$ is one-one and onto. Hence, find f^{-1}.

Section B

5. Find the domain of f(x) = $\sin^{-1}(-x^2)$.
6. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)+\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.
7. Evaluate $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)+\frac{\pi}{6}\right]$

Section C

8. Construct 2 $\times$ 3 matrix whose element aij are given by $aij=\begin{bmatrix}2i+j & when & i<j\\ 4i.j & when & i=j\\ i+2j & when & i>j\end{bmatrix}$
9. Express the matrix $A=\begin{bmatrix}2 & 4 & -6\\ 7 & 3 & 5\\ 1 & -2 & 4\end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix.
10. Find the matrix A satisfying the matrix equation:
    $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix} A \begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Section D

11. Use the product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations x+ 3z = - 9, - x + 2y - 2z = 4 , 2x - 3y + 4z = -3.
12. Using matrix method, solve the system of equations
    2x - 3y + 5z = 11;
    3x + 2y - 4z = -5;
    x + y - 2z = -3.

13. Prove that $\Delta = \begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = \left(1-x^2\right)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$

Section E

14. If x = $\tan\left(\frac{1}{a}\log y\right)$, then show that $\left(1+x^2\right)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$.

15. If y = $e^{m\sin^{-1}x}$, then show that $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$.

16. If x = a (1 + cosθ), y = a (θ + sinθ), prove that $\frac{d^2y}{dx^2} = \frac{-1}{a}$ at $\theta = \frac{\pi}{2}$

17. If y = $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, then show that $\left(1-x^2\right)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0$.

18. Find the value of k so that the function f is continuous at the indicated point: $f(x) = \begin{cases} \dfrac{1-\cos kx}{x\sin x}, & if\ x \neq 0 \\ \dfrac{1}{2}, & if\ x = 0 \end{cases}$ at x = 0

19. If $x^{13}y^7 = (x+y)^{20}$, prove that $\frac{dy}{dx} = \frac{y}{x}$

Section F

20. Find the difference between the greatest and least values of the function f(x) = sin2x - x, on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

21. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi vertical angle is $\tan^{-1}(0.5)$ water is poured into it at a constant rate of 5 m^3/hr. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.

22. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$ Also find the maximum volume.

23. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

24. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

Section G

25. Evaluate: $\int \frac{(x^2+1)}{(x^4+x^2+1)}dx$

26. Evaluate the integral: $\int \sqrt{cosecx - 1}\,dx$

27. Evaluate: $\int \sqrt{\frac{a-x}{a+x}}dx$

28. Evaluate $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{\frac{5}{2}}}dx$

29. Evaluate the integral: $\int \frac{1}{\cos(x-a)\cos(x-b)}dx$

30. Prove that: $\int_0^{\pi} \frac{x\tan x}{(\sec x \csc x)}dx = \frac{\pi^2}{4}$

31. Evaluate: $\int_0^{\pi/2} \frac{1}{2\cos x + 4\sin x}dx$

32. Evaluate $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right]dx.$

Section H

33. Solve the following diff. eq. $\left(x^2+1\right)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$

34. Solve the diff. eq $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

Section I

35. Solve the following linear programming problem graphically.

Minimize Z = 200 x + 500 y

Subject to constraints

$x + 2y \geq 10$

$3x + 4y \leq 24$

and $x \geq 0, y \geq 0$

36. Minimize and Maximize Z = x + 2y subject to $x + 2y \geqslant 100, 2x - y \leqslant 0, 2x + y \leqslant 200, x, y \geqslant 0$.